TEEGALA KRISHNA REDDY ENGINEERING COLLEGE

(UGC-Autonomous)

Approved by AICTE, Affiliated by JNTUH, Accredited by NAAC- 'A' Grade

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DEPARTMENT OF CIVIL ENGINEERING
LECTURE NOTES

ACADEMIC YEAR: 2023-24

REGULATION: R20

YEAR/SEMESTER: III - I

SUBJECT: **STRUCTURALANALYSIS– II**

SUBJECT CODE: **20CE5PC04**

FACULTY NAME: P.CHAITANYA

SYLLABUS:

UNIT – I

Moment Distribution Method – Analysis of continuous beams with and without settlement of supports using –Analysis of Single Bay Single Storey Portal Frames including side Sway
Analysisofinclinedframes–
ShearforceandBendingmomentdiagrams,Elastic curve.

Kani's Method: Analysis of continuous beams including settlement of supports–Analysis of single bay single storey and single bay two Storey Frames including Side Sway using Kani's Method – Shear force and bending moment diagrams – Elastic curve.

STRUCTURAL ANALYSIS - II

Unit - 01

Two Hinged Arches and Moment Distribution method :-

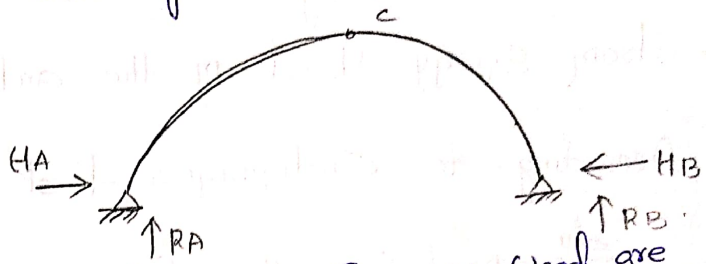
Mainly three types of arches are used in practice they are Three - Hinged, Two - Hinged and Hingeless arches.

Analysis of Two - Hinged Arches :-

Two Hinged Arch is the Statically Indeterminate Structure to degree One.

$$D_S = r_1 - 3$$

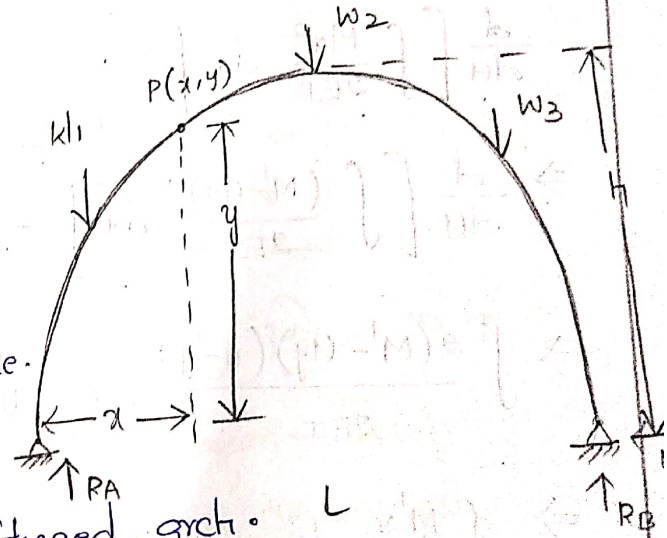
$$\Rightarrow 4 - 3 = 1$$



→ The different Shapes of 2- Hinged arches Commonly Used are Circular and Parabolic.

→ In the case of two - Hinged arch, we have four Unknown reactions, H_A, H_B, R_A & R_B. But there are only three Equations of Equilibrium available.

Hence the degree of Statical Indeterminancy is "One", for two Hinged arch.



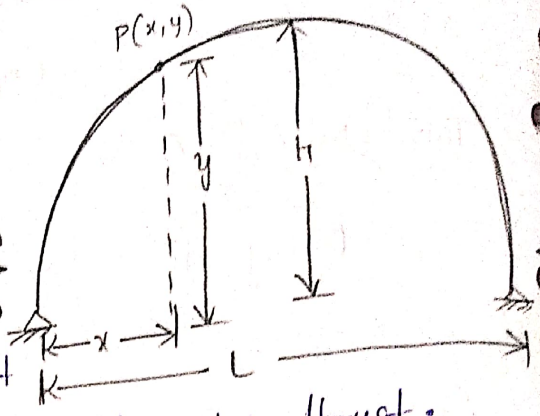
→ Normally Arches are Subjected to Vertical forces Only. Hence Equilibrium of forces in the Horizontal directions. gives $\boxed{H_A = H_B = H}$

→ The Horizontal force 'H' may be taken as the redundant force. The Expression for Horizontal thrust may be found by any one of the following two methods.

(i) Castigliano's first Theorem (Strain Energy method)

(ii) Unit load method.

Castigliano's first theorem (Strain Energy method):



Consider the arch shown in figure. Let M_x be the Bending moment at point P(x,y) on the arch.

→ Support A being taken as Origin

Let M' be the Beam moment (moment due to applied loads) at p and H be the Horizontal thrust.

$$M_x = M' - Hy.$$

∴ Strain Energy Stored in the arch $U = \int \frac{M_x^2}{2EI} dx.$

According to Castigliano's first theorem.

$$\frac{du}{dH} = 0 \quad [\text{Since the Supports are on yielding}].$$

$$\frac{d}{dH}\left[\int \frac{M_x^2}{2EI} \cdot dx\right]$$

$$\Rightarrow \frac{d}{dH}\left[\int \frac{(M'-Hy)^2}{2EI} \cdot dx\right] = 0$$

$$\Rightarrow \int \frac{2(M'-Hy)(0-y)}{2EI} \cdot dx = 0$$

$$\Rightarrow \int \frac{M'y}{EI} \cdot dx + \int \frac{Hy^2}{EI} \cdot dx = 0$$

$$H = \frac{\int \frac{M'y}{EI} dx}{\int \frac{y^2}{EI} \cdot dx} \Rightarrow \boxed{H = \frac{\int M'y \cdot dx}{\int y^2 \cdot dx}}$$

Analysis of Parabolic Arches:-

Consider a Parabolic arch as shown in figure.

Taking the left Support or left Springing Point as the Origin

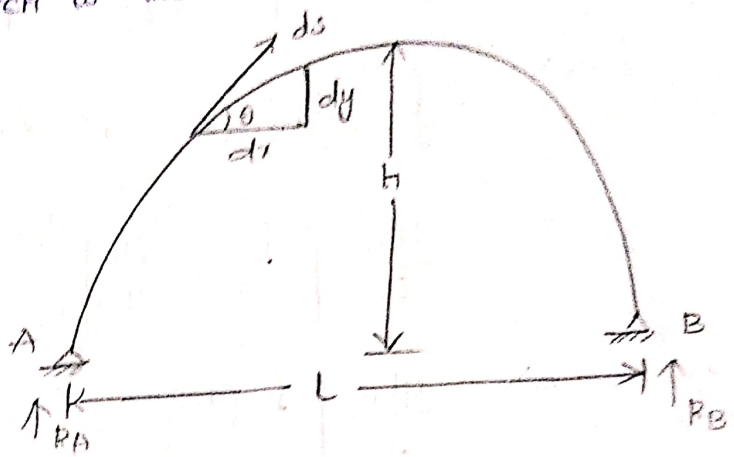
The Equation of the Parabola is given,

$$y = \frac{4hx(l-x)}{l^2}$$

Where, 'l' is the Span and 'h' is the rise.

Let P(x,y) be a Point on the arch and ds is the Elemental length.

→ Let 'θ' be the slope of the arch to the Horizontal at P.



Then $\cos\theta = \frac{dx}{ds}$

$ds = dx \sec\theta$

$$H = \frac{\int M' \cdot y \, ds/EI}{\int y^2 \cdot \frac{ds}{EI}}$$

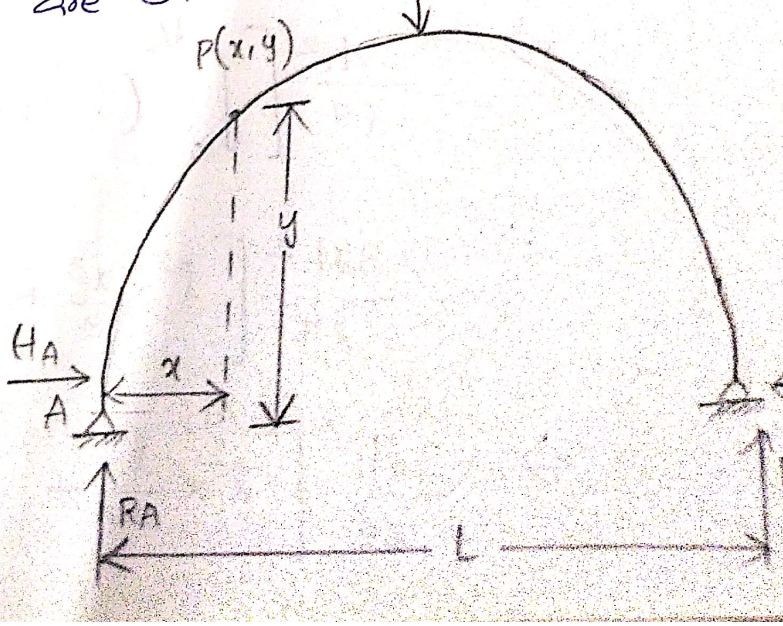
$$\Rightarrow \frac{\int M' \cdot y \cdot \frac{dx \sec\theta}{EI}}{\int M'^2 y^2 \cdot \frac{dx \sec\theta}{EI}}$$

$$\Rightarrow \boxed{H = \frac{\int M' \cdot y \cdot dx}{\int y^2 \cdot dx}}$$

Q:- A Two-Hinged Parabolic arch of Span 'l' and rise 'h' Carries a Concentrated 'W' at the Crown. Determine the Expression for Horizontal thrust developed at Springings.

Dt:- 15/09/22

Consider a Typical two-Hinged arch as Shown in figure. Due to Symmetry reactions A and B are Same and Equal to W/2.



Step (1):- Support Reactions

$R_A + R_B = W$

$R_A = R_B = W/2$

- The Beam moment at point $P(x,y)$ is given by $M' = \frac{W}{2} \times x$
- The Horizontal Thrust $H = \frac{\int M' \cdot y \cdot dx}{\int y^2 \cdot dx}$

$$\int M' \cdot y \cdot dx = \int \left(\frac{W}{2} x\right)\left(\frac{4hx(l-x)}{l^2}\right) \cdot dx$$

$$\Rightarrow 2\int_0^{l/2} \frac{W}{2} x \left[\frac{4hxl - 4hx^2}{l^2}\right] dx$$

$$\Rightarrow 2\int_0^{l/2} \frac{4hw}{2l^2} (lx^2 - x^3)\, dx$$

$$\Rightarrow \frac{4hw}{l^2}\left[\frac{lx^3}{3} - \frac{x^4}{4}\right]_0^{l/2}$$

$$\Rightarrow \frac{4hw}{l^2}\left[\frac{lx(1/2)^3}{3} - \frac{(l/2)^4}{4}\right] \Rightarrow \frac{4hw}{l^2}\left[\frac{l^4}{24} - \frac{l^4}{64}\right]$$

$$\Rightarrow \frac{4hw}{l^2}\left[\frac{5}{192} l^4\right] \Rightarrow \frac{20hwl^2}{192}$$

$$= \frac{5whl^2}{48}.$$

(ii) $$\int y^2 \cdot dx = \int \left[\frac{4hx(l-x)}{l^2}\right]^2 dx.$$

$$\Rightarrow 2\int_0^{l/2} \left[\frac{16h^2x^2}{l^4}(l^2 + x^2 - 2lx)\right] dx.$$

$$\Rightarrow \frac{32h^2}{l^4}\left[\int_0^{l/2} (l^2x^2 + x^4 - 2lx^3)\, dx\right]$$

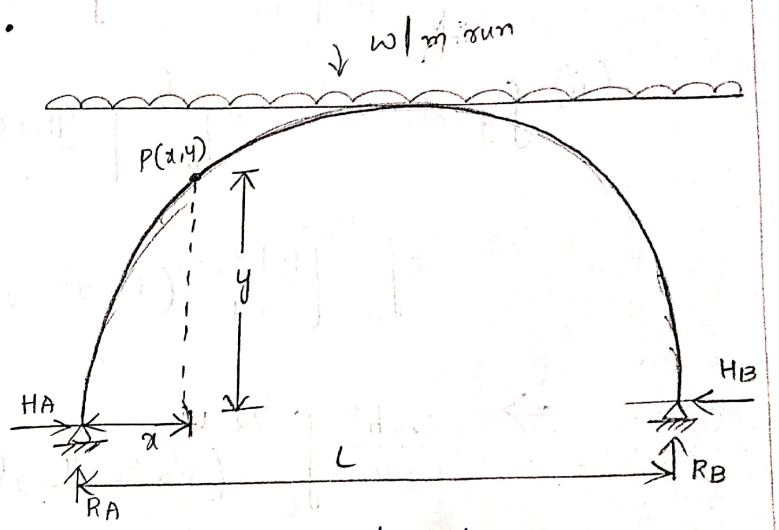
$$\Rightarrow \frac{32h^2}{l^4}\left[l^2 \times \frac{x^3}{3} + \frac{x^5}{5} - \frac{2lx^4}{4}\right]_0^{l/2}$$

$$\Rightarrow \frac{32h^2}{l^4}\left[l^2 \times \frac{l^3}{24} + \frac{l^5}{160} - \frac{2l^5}{64}\right] \Rightarrow \frac{8}{15} h^2 l$$

$$H = \frac{\int M' \cdot y \cdot dx}{\int y^2 \cdot dx} \Rightarrow \frac{\frac{5wlh\ell^2}{48}}{\frac{8}{15}h^2 \times \ell} \Rightarrow \frac{5wlh\ell^2}{48} \times \frac{15}{8h^2 \times \ell}$$

$$H = \frac{25wl}{128h} \text{ //.}$$

Q:- A Two Hinged Parabolic Arch of Span 'l' and rise 'h' Carries a udl of w per m run over the whole Span. Assume $I = I_0 \sec\alpha$, find the Expression for the Horizontal thrust develop.



Step 1 :- Support Reactions.

$$R_A + R_B = wl$$

$$\Rightarrow \Sigma M_A = 0$$

$$= -R_B \times \ell + w \times l \times \frac{\ell}{2} = 0$$

$$\Rightarrow R_B = \frac{wl}{2} \text{ and } R_A = \frac{wl}{2}$$

Step 2 :- The Beam moment at Point P(x,y) is given by.

$$M' = \left[\frac{wl}{2} \times x\right] - \left[\frac{wx^2}{2}\right]$$

The Horizontal Thrust $H = \dfrac{\int M' \cdot y \cdot dx}{\int y^2 \cdot dx}$.

$$\Rightarrow \int M' \cdot y \cdot dx \Rightarrow \int \left[\left(\frac{wl}{2}x\right) - \left(\frac{wx^2}{2}\right)\left(\frac{4hx(l-x)}{l^2}\right)\right] dx.$$

$$\Rightarrow 2\int_0^{l/2} \left[\frac{wlx - wx^2}{2}\right]\left[\frac{4hx(l-x)}{l^2}\right] dx.$$

$$\Rightarrow 2\int_0^{l/2} \left[\frac{wlx - wx^2}{2}\right]\left[\frac{4hlx - 4hx^2}{l^2}\right] dx.$$

$$\Rightarrow \frac{1}{l^2}\int_0^{l/2} \left(wl^2x^2 4h - 4hx^3 lw - 4hlx^3 w + 4hx^4 w\right) dx.$$

$$\frac{1}{l^2}\left[\frac{4wl^2x^3h}{3} - \frac{8hlwx^4}{4} + \frac{4hwx^5}{5}\right]_0^{l/2}$$

$$\Rightarrow \frac{1}{l^2}\left[\frac{4wl^2(l/2)^3h}{3} - \frac{8whl(l/2)^4}{4} + \frac{4wh(l/2)^5}{5}\right]$$

$$\Rightarrow \frac{1}{l^2}\left[\frac{4wl^5h}{24} - \frac{8whl^5}{64} + \frac{4whl^5}{160}\right]$$

$$\Rightarrow \frac{1}{l^2}\left[\frac{1}{15}whl^3\right] \Rightarrow \frac{1}{15}whl^3.$$

(ii) $\int y^2 \cdot dx \Rightarrow 2\int_0^{l/2}\left[\frac{4hx(l-x)}{l^2}\right]^2 dx$

$$\Rightarrow 2\int_0^{l/2}\left[\frac{16h^2x^2}{l^4}(l^2+x^2-2lx)\right]dx.$$

$$\Rightarrow \frac{32h^2}{l^4}\int_0^{l/2}(l^2x^2+x^4-2lx^3)\,dx.$$

$$\Rightarrow \frac{32h^2}{l^4}\left[l^2x\frac{x^3}{3}+\frac{x^5}{5}-\frac{2lx^4}{4}\right]_0^{l/2}$$

$$\Rightarrow \frac{32h^2}{l^4}\left[l^2x\frac{l^3}{24}+\frac{l^5}{160}-\frac{2l^5}{64}\right]$$

$$\Rightarrow \frac{32h^2l^5}{60l^4} = \frac{8}{15}h^2l$$

$$H = \frac{\int My\,dx}{\int y^2\cdot dx} = \frac{\frac{wl^3}{15}h}{\frac{8}{15}h^2l} \Rightarrow \frac{wl^2}{8h}$$

Note:- If only Half of the Span is loaded with udl then the Horizontal thrust developed in the arch is $H = \frac{1}{2} \times \frac{wl^2}{8h}$

$$\boxed{H = \frac{wl^2}{16h}}$$

(3) Show that the Horizontal Thrust developed in a Parabolic arch of Span 'l' and rise 'h' Subjected to a Concentrated load 'W' at a distance 'a' from a Springing is given by $H = \frac{5}{8}\frac{W}{hl^3}a(l-a)(l^2+la-a^2)$.

Sol:- Step ① :- Support Reactions :

$R_A + R_B = 2W$

Therefore $R_A = R_B = W$.

Step ② : Beam moment @ any Point P(x, y) is

$M' = Wx$ limits 0 to 'a'

$M' = Wx - W(x-a)$

$= Wa$ [limits a to $l/2$]

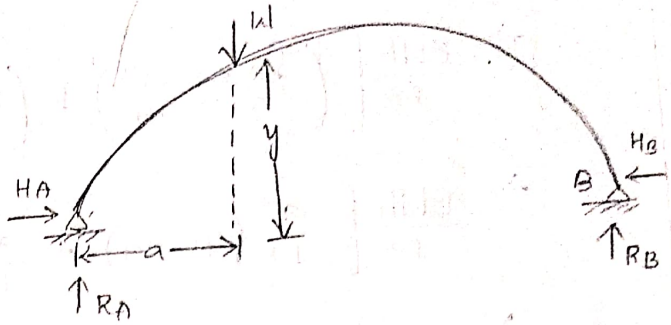
$\therefore 2H = \frac{\int M' \cdot y \cdot dx}{\int y^2 \cdot dx}$.

$y = \frac{4hx(l-x)}{l^2}$

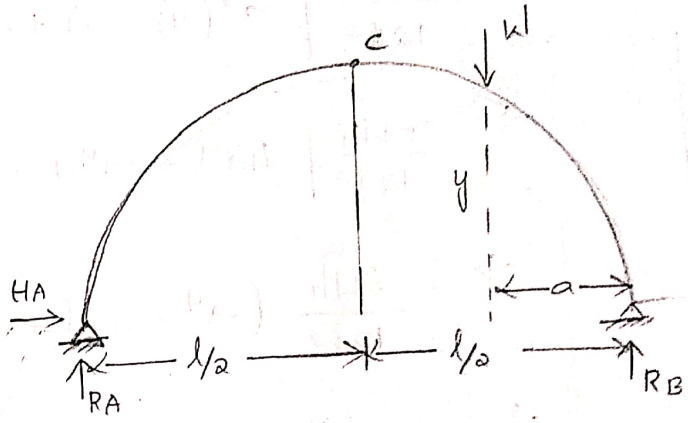
$\therefore \int M' \cdot y \cdot dx =$

$2\left[\int_0^a Wx \cdot y \cdot dx + \int_a^{l/2} W \cdot a \cdot y\, dx\right]$

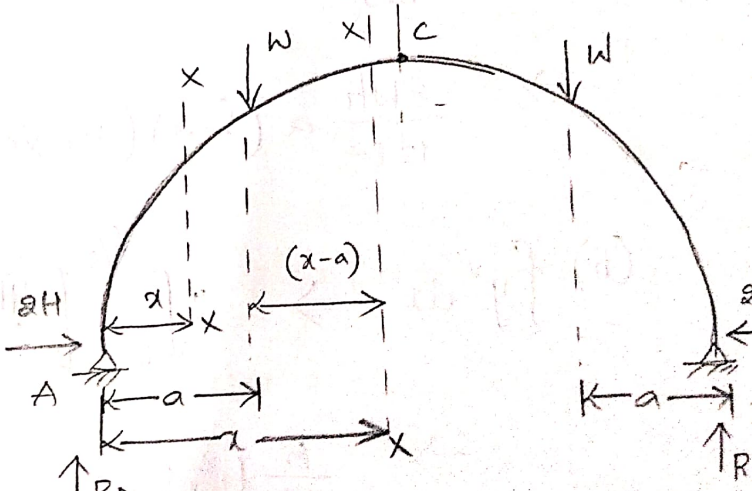
Case ①



Case (2)



Case ③



$\Rightarrow 2\int_0^a Wx \cdot \frac{4hx(l-x)}{l^2}dx + \int_a^{l/2} W \cdot ax \frac{4hx(l-x)}{l^2} \cdot dx.$

$\Rightarrow 2\left[\int_0^a Wx \frac{4hx(l-x)}{l^2}dx + \int_a^{l/2} Wax \frac{4hx(l-x)}{l^2}dx\right]$

$\Rightarrow \frac{8Wh}{l^2}\left[\int_0^a (lx^2 - x^3)dx + a\int_a^{l/2}(lx - x^2)dx\right]$

$$\frac{8Wh}{l^2}\left[\left(\frac{lx^3}{3}-\frac{x^4}{4}\right)_0^a + a\left(\frac{lx^2}{2}-\frac{x^3}{3}\right)_a^{l/2}\right]$$

$$\Rightarrow \frac{8Wh}{l^2}\left[\left(\frac{la^3}{3}-\frac{a^4}{4}\right)+a\left[\left(\frac{lxl^2}{8}-\frac{l^3}{24}\right)-\left(\frac{la^2}{2}-\frac{a^3}{3}\right)\right]\right]$$

$$\Rightarrow \frac{8Wh}{l^2}\left[\left(\frac{la^3}{3}-\frac{a^4}{4}\right)+\left(\frac{al^3}{8}-\frac{al^3}{24}\right)-\left(\frac{la^3}{2}-\frac{a^4}{3}\right)\right]$$

$$\Rightarrow \frac{8Wh}{l^2}\left[\frac{a^3}{12}(4l-3a)+\frac{al^3}{12}-\frac{a^3}{6}(3l-2a)\right]$$

$$\Rightarrow \frac{8Wh}{12l^2}\left[a^3(4l-3a)+al^3-2a^3(3l-2a)\right]$$

$$\Rightarrow \frac{8Wh}{12l^2}\left[4a^3l-3a^4+al^3-6a^3l+4a^4\right]$$

$$\Rightarrow \frac{8Wh}{12l^2}\left(a^4-2a^3l+al^3\right)$$

$$\Rightarrow \frac{8Wh}{12l^2}a\left(a^3-2a^2l+l^3\right)$$

$$\Rightarrow \frac{8Wh}{12l^2}a(l-a)\left(l^2+la-a^2\right)$$

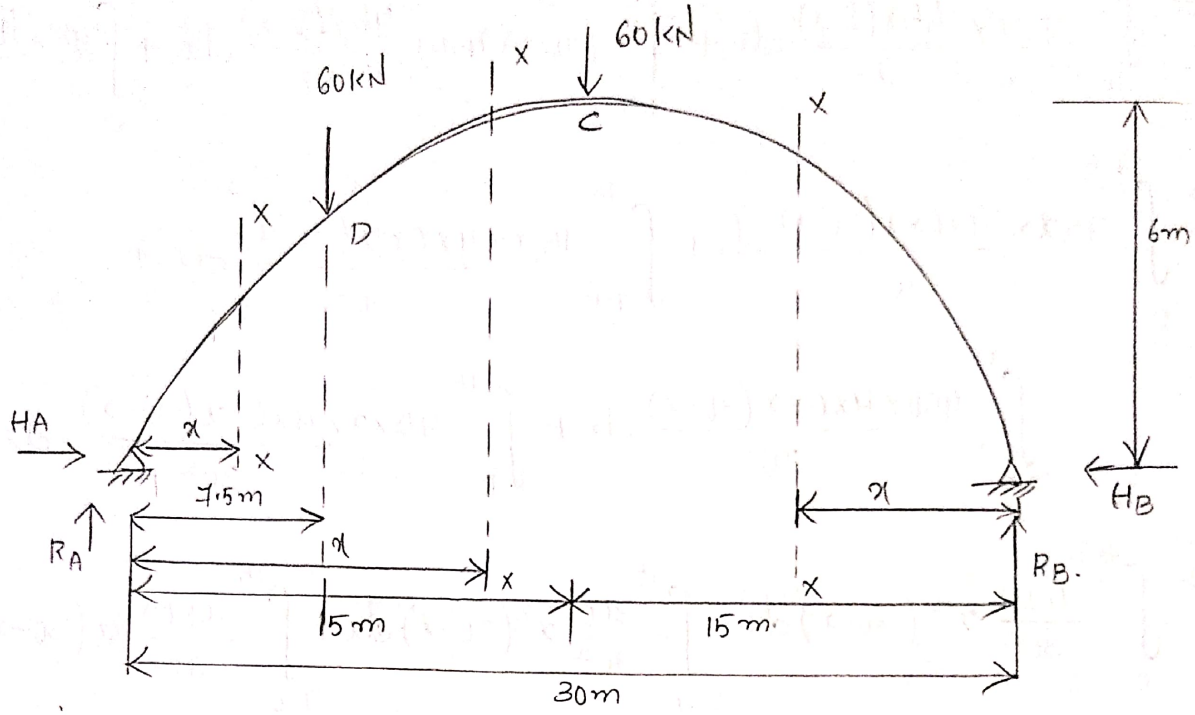
(ii) $\int y^2\cdot dx \Rightarrow 2\int_0^{l/2}\left[\frac{4hx(l-x)}{l^2}\right]^2 dx$

$$\Rightarrow \frac{8}{15}h^2xl$$

$$H=\frac{\int M\cdot y\cdot dx}{\int y^2\cdot dx} \Rightarrow \frac{\frac{8Wh}{12l^2}a(l-a)\left(l^2+la-a^2\right)}{\frac{8}{15}h^2xl}$$

$$\Rightarrow \frac{5}{8}\frac{W}{hl^3}a(l-a)\left(l^2+la-a^2\right)$$ "

Q:- A Two-hinged Parabolic arch (5) of Span 30 m and Dt:- 21/09/22
rise 6m Carries two point loads, Each 60 kN, acting at 7.5 m and 15 m from the left End respectively. The moment of Inertia I Varies as the Secant of Slope. Determine the Horizontal Thrust, maximum Positive and negative moments in the arch.



Step ①: Support Reactions

$$R_A + R_B = 120 \text{ kN}$$

$$\Sigma M_A = 0 \Rightarrow -R_B \times 30 + 60 \times 15 + 60 \times 7.5 = 0$$

$$R_B = 45 \text{ kN} \quad \text{and} \quad R_A = 75 \text{ kN}.$$

Step ②: Beam moment at any Point P(x,y) is

$M' = 75x$ limits 0 to 7.5 — (AD Portion)

$M' = 75x - 60(x - 7.5)$ — (DC Portion).

$= 75x - 60x + 450 \Rightarrow 15x + 450$ (limits 7.5 to 15)

$M' = 45x$ (limits 0 to 15) (BC Portion)

$$H = \frac{\int M' \cdot y \cdot dx}{\int y^2 \cdot dx}$$

$$\int M^1 \cdot y\,dx \Rightarrow \left[\int_0^{7.5} 75x + \int_{7.5}^{15} (15x+450) + \int_0^{15} 45x \right] y \cdot dx$$

$$\Rightarrow \int_0^{7.5} 75xy \cdot dx + \int_{7.5}^{15} (15x+450)\,y\,dx + \int_0^{15} 45x \cdot y \cdot dx$$

$$\Rightarrow \int_0^{7.5} 75x \times \frac{4hx(l-x)}{l^2}\,dx + \int_{7.5}^{15} (15x+450) \frac{4hx(l-x)}{l^2}\,dx + \int_0^{15} 45x \frac{4hx(l-x)}{l^2}\,dx$$

$$\Rightarrow \int_0^{7.5} 75x \times \frac{4\times6\times x(30-x)}{30^2}\,dx + \int_{7.5}^{15} 15x \times \frac{4\times6\times x(30-x)}{30^2}\,dx +$$

$$\int_{7.5}^{15} 450 \times \frac{4\times6\times x(30-x)}{30^2}\,dx + \int_0^{15} 45\times x \times \frac{4\times6\times x(30-x)}{30^2} \cdot dx$$

$$\Rightarrow \int_0^{7.5} \frac{1800}{30^2} x^2(30-x)\,dx + \int_{7.5}^{15} \frac{360}{30^2} x^2(30-x)\,dx + \int_{7.5}^{15} \frac{10800}{30^2} x(30-x)\,dx +$$

$$\int_0^{15} \frac{1080}{30^2} x^2(30-x)\,dx$$

$$\Rightarrow \frac{1}{30^2}\left[\int_0^{7.5} 1800x^2(30-x)\,dx + \int_{7.5}^{15} 360\,x^2(30-x)\,dx + \int_{7.5}^{15} 10800x(30-x) + \right.$$

$$\left. \int_0^{15} 1080x^2(30-x)\,dx \right.$$

$$\Rightarrow \frac{1}{30^2}\left[1800 \int_0^{7.5} x^2(30-x)\,dx + 360 \int_{7.5}^{15} x^2(30-x)\,dx + 10800 \int_{7.5}^{15} x(30-x)\,dx \right.$$

$$\left. + 1080 \int_0^{15} x^2(30-x)\,dx \right.$$

$$\Rightarrow \frac{1}{30^2}\left[1800 \left(\frac{30x^3}{3} - \frac{x^4}{4} \right)_0^{7.5} + 360 \left(\frac{30x^3}{3} - \frac{x^4}{4} \right)_{7.5}^{15} + 10800 \left(\frac{30x^2}{2} - \frac{x^3}{3} \right)_{7.5}^{15} \right.$$

$$\left. 1080 \left(\frac{30x^3}{3} - \frac{x^4}{4} \right)_0^{15} \right]$$

$$\Rightarrow \frac{1}{30^2}\left[1800\left(\frac{30\times7.5^3}{3}-\frac{7.5^4}{4}\right)+360\left(\frac{30\times15^3}{3}-\frac{15^4}{4}-\frac{30\times7.5^3}{3}+\frac{7.5^4}{4}\right)+\right.$$

$$\left.10800\left(\frac{30\times15^2}{2}-\frac{15^3}{3}-\frac{30\times7.5^2}{2}+\frac{7.5^3}{3}\right)+\left(1080\left(\frac{30\times15^3}{3}-\frac{15^4}{4}\right)\right)\right]$$

$$\Rightarrow \frac{1}{30^2}\left[1800\left(\frac{219375}{64}\right)+360\left(\frac{1130625}{64}\right)+10800\left(\frac{12375}{8}\right)+1080\left(\frac{84375}{4}\right)\right]$$

$$\Rightarrow \quad 57796.875 \text{ kN.m}$$

$$\int y^2 \cdot dx \Rightarrow \int_0^{30}\left[\frac{4hx(l-x)}{l^2}\right]^2 \cdot dx$$

$$\Rightarrow \frac{8}{15}h^2xl \Rightarrow \frac{8}{15}\times6^2\times30 = 576 \text{ m}.$$

The Horizontal ~~Hroust~~ $H = \dfrac{\int M^1 \cdot y \cdot dx}{\int y^2 \cdot dx} = \dfrac{57796.875}{576}$

$$\Rightarrow \boxed{H = 100.34 \text{ kN}}$$

Step ③: Calculate Maximum Positive and Negative BM:-

The maximum Positive BM occurs under the loads.

① The B.M at a distance of 7.5 m.

$$y = \frac{4hx}{l^2}(l-x) \Rightarrow \frac{4\times6\times7.5}{30^2}(30-7.5)$$

$$y = 4.5 \text{ m}$$

$$\therefore M_x = 75x - Hxy$$

$$= 75\times7.5 - 100.34\times4.5$$

$$= 112.5 \text{ kN.m}$$

② The B.M at a distance of 15 m.

$$y = \frac{4Hx(l-x)}{l^2} = \frac{4\times6\times15}{30^2}(30-x)$$

$$y = 6m.$$

$$M_x = R_B \times x - H \times y$$

$$= 45\times15 - 100\times6$$

$$= 75 \text{ KN.m}.$$

Maximum Negative moment Occurs in right half Portion i.e 'x' from Support 'B'.

$$M_x = 45x - 100\times y \Rightarrow 45x - 100\times\frac{4Hx(l-x)}{l^2}$$

$$\frac{dM_x}{dx} = 0 \Rightarrow \frac{d}{dx}\left[45x - \frac{100\times4\times6\times x(30-x)}{30^2}\right]$$

$$\Rightarrow \frac{d}{dx}\left[45x - \frac{100\times24x}{30^2}(30-x)\right] \Rightarrow \frac{d}{dx}\left[45x - \frac{720x - 24x^2}{30^2}\right]$$

$$\Rightarrow \frac{d}{dx}\left[45x - \frac{200}{75}x(30-x)\right] \Rightarrow \frac{d}{dx}\left[45x - \frac{200}{75}(30x-x^2)\right]$$

$$\Rightarrow 45 - \frac{200}{75}[30-2x] = 0$$

$$\Rightarrow x = 6.56m.$$

Substitue the Value of 'x' in 'M_x' equation.

$$M_x = 45x - \frac{100\times4\times6\times x(l-x)}{l^2}$$

$$= 45\times6.56 - \frac{100\times4\times6\times6.56(30-6.56)}{30^2}$$

$$\boxed{M_x = -115.35 \text{ KN.m}}$$

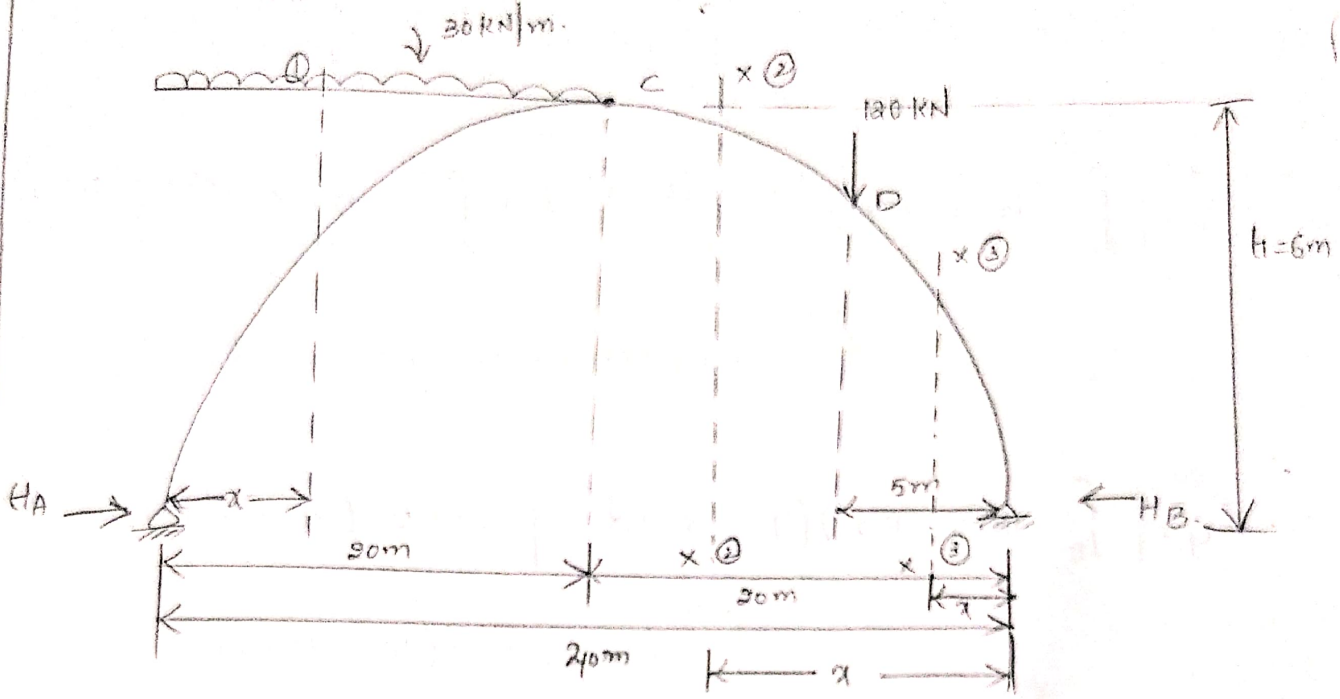
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5) A Two-Hinged Parabolic ① arch as shown in figure. Determine

(i) Horizontal Thrust

(ii) Maximum Positive and Negative moments

(iii) Shear force and Normal thrust at 10m from right Support. Assume $I = I_0 \sec\theta$. Where I_0 is the moment of Inertia at the Crown and θ is the Slope at the Section Under Consideration.



Step ①: Support Reactions

$R_A + R_B = 720$ kN

$\Sigma M_A = 0 \Rightarrow -R_B \times 40 + 120 \times 35 + 30 \times 20 \times \frac{20}{2}$

$R_B = 255$ kN and $R_A = 465$ kN.

Step ②: Beam moment at any Point P(x, y).

$M_x = R_A \times x - 30 \times x \times \frac{x}{2}$ — (AC Portion)

$= 465x - \frac{30x^2}{2}$ Limits (0 to 20)

$M_x = R_B \times x$ — (BD Portion)

$= 255x$ Limits (0 to 5)

$M_x = R_B \times x - 120(x-5)$ — (CD Portion).

$\Rightarrow 255x - 120x + 600$

$= 135x + 600$ (limits 5 to 20).

Step ③: Calculation of Horizontal Thrust :-

$$\int M' \cdot y \cdot dx$$

$$\Rightarrow \int_0^{20} (465x - 15x^2) \times \frac{4Hx(l-x)}{l^2}\, dx + \int_0^{5} 255x \times \frac{4Hx(l-x)}{l^2}\, dx + \int_5^{20} (135x + 600) \times \frac{4Hx(l-x)}{12}\, dx.$$

$$\Rightarrow \frac{1}{40^2}\left[\int_0^{20} (465x - 15x^2) \frac{4\times 6 \times x(40-x)}{1}\, dx + \int_0^{5} 255x \times 4\times 6\times x(40-x)\, dx + \int_5^{20} (135x + 600) \times 4\times 6\times x\,(40-x)\, dx.\right.$$

$$\Rightarrow \frac{24}{40^2}\left[\int_0^{20} (465x - 15x^2)(40x - x^2)\, dx + \int_0^{5} 255x^2(40-x)\, dx + \int_5^{20} (135x + 600)(40x - x^2)\, dx.\right.$$

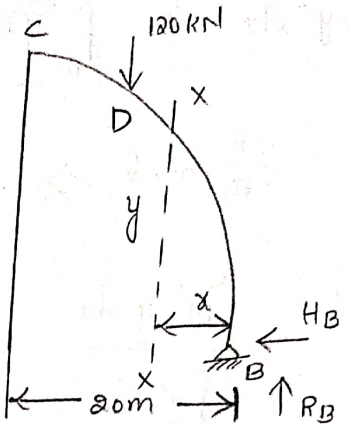
$$\Rightarrow \frac{24}{40^2}\left[\int_0^{20} (18600x^2 - 465x^3 - 600x^3 + 15x^4)\, dx + \int_0^{5} (10200x^2 - 255x^3)\, dx + \int_5^{20} (5400x^2 - 135x^3 + 24000x - 600x^2)\, dx\right.$$

$$\Rightarrow \frac{24}{40^2}\left[\left(\frac{18600x^3}{3} - \frac{465x^4}{4} - \frac{600x^4}{4} + \frac{15x^5}{5}\right)_0^{20} + \left(\frac{10200x^3}{3} - \frac{255x^4}{4}\right)_0^{5} + \left(\frac{5400x^3}{3} - \frac{135x^4}{4} + \frac{24000x^2}{2} - \frac{600x^3}{3}\right)_5^{20}\right.$$

$$\Rightarrow \frac{24}{40^2}\left[16.6\times 10^6 + 385156.25 + 12.2\times 10^6 - 478.90\times 10^3\right]$$

$$\Rightarrow 430593.8438 \text{ KN}\cdot\text{m}.$$

Consider Portion 'CB' for Minimum.



(a) BD Portion :-

$$M_x = R_B \times x - H_B \times y$$

$$= 255x - 560.66 \times \frac{4hx(l-x)}{l^2}$$

$$= 255x - 560.66 \times \frac{4 \times 6 \times x(40-x)}{40^2}$$

$$= 255x - \frac{560.66 \times 24}{40^2}[40x - x^2]$$

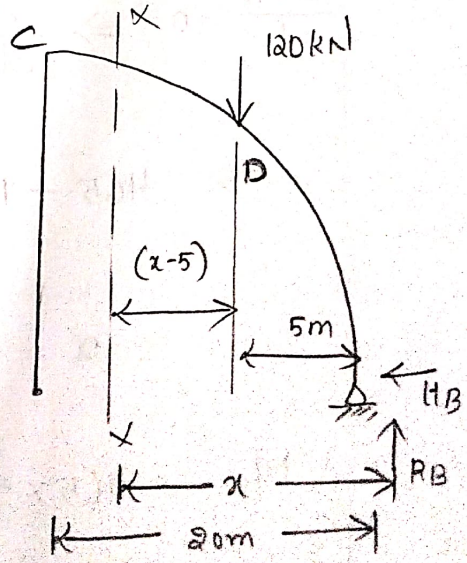
$$\frac{dM_x}{dx} = 0 \Rightarrow \frac{d}{dx}\left[255x - \frac{560.66 \times 24}{40^2}(40x - x^2)\right] = 0$$

$$\Rightarrow \left[255 - \frac{560.66 \times 24}{40^2}(40 - 2x)\right] = 0$$

$$x = 4.83 \text{ m}.$$

$$M_x = 225[4.83] - \frac{560.66 \times 24}{40^2}[40 \times 4.83 - 4.83^2]$$

$$M_x = -196.94 \text{ kN.m}.$$



(b) CD Portion :-

$$M_x = R_B \times x - H_B \times y - 120(x-5)$$

$$= 255x - 560.66 \times \frac{4 \times 6 \times x(40-x)}{40^2} - 120(x-5)$$

$$255x - \frac{560.66 \times 24}{40^2}(40x - x^2) - 120x - 600.$$

$$\frac{dM_x}{dx} = 0 \quad \frac{d}{dx}\left[255x - \frac{560.66 \times 24}{40^2}(40x - x^2) - 120x - 600\right]$$

$$\Rightarrow 255 - \frac{560.66 \times 24}{40^2}(40 - 2x) - 120 = 0$$

(ii) $\int y^2 \cdot dx \Rightarrow \int_0^{40} \left[\frac{4hx(l-x)}{l^2}\right]^2 \cdot dx \Rightarrow \int_0^{40} \left[\frac{4\times 6\times x(40-x)}{40^2}\right]^2 dx$

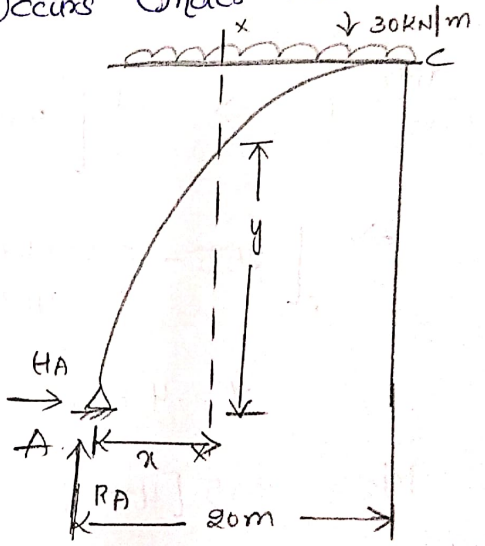
$\Rightarrow \frac{8}{15} h^2 \times l \Rightarrow \frac{8}{15} \times 6^2 \times 40 = 768 \cdot m.$

$\Rightarrow H = \frac{\int M' \cdot y \cdot dx}{\int y^2 \cdot dx} \Rightarrow \frac{430593.843}{768} = 560.66 \text{ KN}.$

Step ③ : Calculate the Positive and Negative BM.

The Maximum Bending moment occurs under the loads.

So lets, Consider AC Portion.



$M_x = R_A \times x - 30 \times x \times \frac{x}{2} - H_A \times y.$

$= 465x - \frac{30x^2}{2} - 560.66 \times \frac{4 \times h \times x(l-x)}{l^2}$

$= 465x - 15x^2 - \frac{560.66 \times 24}{40^2}\left[x(40-x)\right]$

$\frac{dM_x}{dx} = 0 \Rightarrow \frac{d}{dx}\left[465x - 15x^2 - \frac{560.66 \times 24}{40^2}\left(40x - 2x^2\right)\right]$

$\Rightarrow 465 - 15 \times 2x - \frac{560.66 \times 24}{40^2}(40 - 2x) = 0$

$x = 9.75 m.$

$\therefore M_x = 465 \times 9.75 - \frac{30 \times 9.75^2}{2} - \frac{560.66 \times 4 \times 24 \times 9.75}{40^2}(40 - 9.75)$

$M_x = 627.54 \text{ kN}.$

$\boxed{M_{max} = 627.54 \text{ kN}.}$

$x = 11.97\ m$.

$$M_x = 255x - \frac{560.66 \times 24}{40^2}(40x - x^2) - 120x - 600$$

$$M_x = 255(11.97) - \frac{560.66 \times 24}{40^2}(40 \times 11.97 - 11.97^2) - 120 \times 11.97 - 600$$

$$M_x = -605.76\ kN.m$$

Therefore, the Maximum Positive moment is 627.54 kN.m at 9.75 m from Left Support and the maximum Negative Moment is −605.76 kN.m at 11.97 m from the right Support.

(4) Shear force and Normal Thrust

At 10m from right Support.

$$= \tan\theta = \frac{dy}{dx}$$

$$\tan\theta = \frac{d}{dx}\left[\frac{4hx}{l^2}(l-x)\right]$$

$$\Rightarrow \tan\theta = \frac{d}{dx}\left[\frac{4h}{l^2}(lx - x^2)\right] \Rightarrow \tan\theta = \left[\frac{4h}{l^2}(l-2x)\right]$$

$$\tan\theta = \left[\frac{4 \times 6}{40^2}(40 - 2 \times 10)\right]$$

$$\theta = \tan^{-1}(0.3),\ \theta \Rightarrow 16^0$$

Normal Thrust $\Rightarrow V\sin\theta + H\cos\theta$

$$\Rightarrow (255 - 120)\sin 16^0 + 560.66 \times \cos 16^0$$

$$= 576.15\ kN.$$

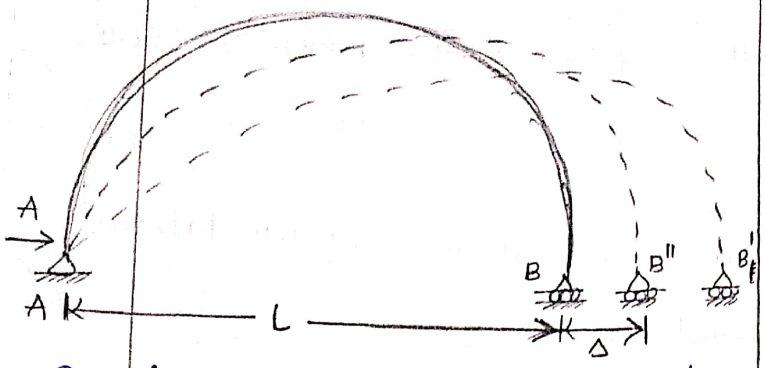
$$Q = V\cos\theta - H\sin\theta$$

$$= 135\cos 16^0 - 560.66 \times \sin 16^0$$

$$= -24.7\ kN\ //.$$

Effect of yielding of Support :-

Dt:- 27/09/22



Consider a two-Hinged arch as shown in figure. the End B of the arch is replaced by the roller Support, due to loads, the Support most Point B'.

Let M' be the beam moment, moment due to Unit horizontal force = 'y'

$$BB' = \int M'y \cdot \frac{ds}{EI}.$$

Let 'H' be the Horizontal force apply to arch. Moment due to this force

$$= Hxy.$$

= moment due to Unit Horizontal force at the Springing (Supports)

$$H = Hxy$$

$$= yx1 = 'y'$$

The Horizontal displacement H,

$$= \int (Hy)\, y \frac{ds}{EI} = \int Hy^2 \frac{ds}{EI}$$

If Supports are yielding by 'Δ'

$$\int Hy^2 \cdot \frac{ds}{EI} = BB' - \Delta$$

$$= \int M'y \frac{ds}{EI} - \Delta$$

$$H = \frac{\int M'y \frac{ds}{EI} - \Delta}{\int y^2 \cdot \frac{ds}{EI}}$$

→ If the arch is Elastic, and the Supports are yield by 'k' due to Horizontal force.

$$\Delta = kH$$

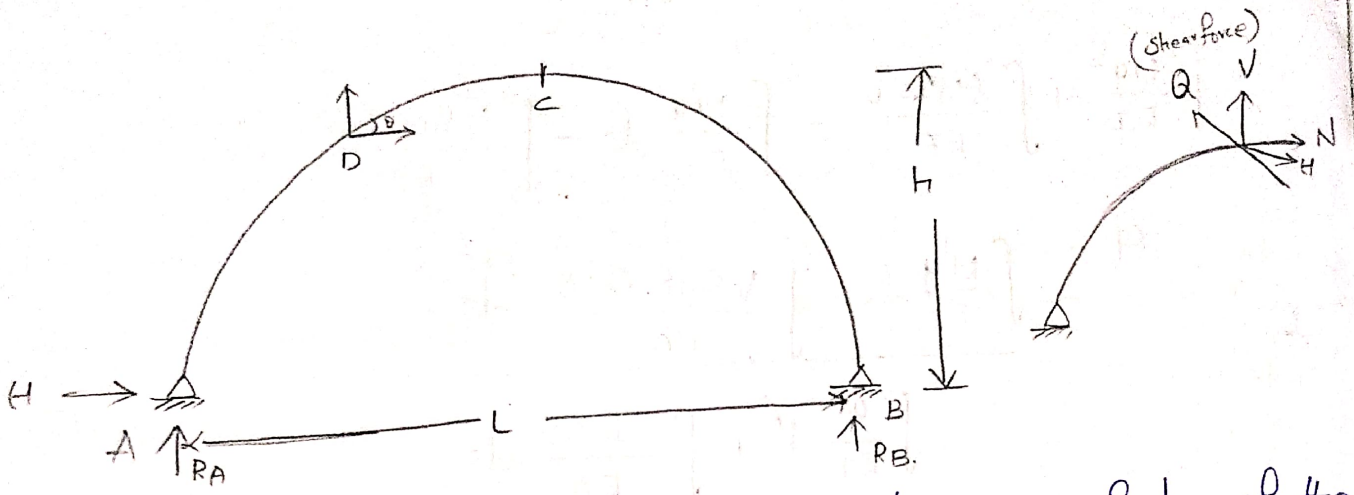
$$\int Hy^2 \cdot \frac{ds}{EI} = \int M'y \frac{ds}{EI} - kH$$

$$H = \frac{\int M'y \frac{ds}{EI}}{\int y^2 \cdot \frac{ds}{EI} + k}$$

$k = \frac{mm}{kN}$

Effect of Shortening of rib :- [rib = Arch]

Dt:- 29/09/22



Consider the arch as shown in figure. The Cross-Section of the arch is subjected to normal thrust also. The arch being made up of Elastic material, Shortening of the rib takes Place. This Shortening reduces the Horizontal thrust developed.

$$N = V \sin\theta + H \cos\theta$$

$$M = M' - Hy$$

The Strain Energy Stored in the arch

$$U = \int \frac{M^2}{2EI}\, ds + \int \frac{N^2}{2EA}\, ds.$$

If Supports are Onyielding, displacement $\frac{dy}{dH} = 0$

$$\therefore \frac{d}{dH}\left[\int \frac{M^2}{2EI}\, ds + \int \frac{N^2}{2EA}\cdot ds\right] = 0$$

$$\Rightarrow \left[\int \frac{2M}{2EI}\cdot\frac{dM}{dH}\cdot ds + \int \frac{2N}{2EA}\cdot\frac{dN}{dH}\cdot ds\right] = 0.$$

$$= \frac{dN}{dH} = \cos\theta \qquad \frac{dM}{dH} = -y.$$

$$\Rightarrow \int \frac{(M' - Hy)}{EI}(-y)\cdot ds + \int \frac{(V\sin\theta + H\cos\theta)}{EI}\cos\theta\cdot ds = 0$$

$$\int \frac{-M'y + Hy^2}{EI} ds + \int \frac{V \sin\theta \cos\theta + H\cos^2\theta}{EA} . ds = 0$$

$$\int \frac{Hy^2}{EI} + \int \frac{H\cos^2\theta}{EA} = \int \frac{M'y}{EI} ds - \int \frac{V \sin\theta \cos\theta}{EA} . ds$$

$$H = \frac{\int \frac{M'y}{EI} ds - \int \frac{V \sin\theta \cos\theta}{EA} . ds}{\int \frac{y^2}{EI} ds + \int \frac{\cos^2\theta}{EA} . ds}$$

Neglecting the Effect of Shear, $V = 0$

$$H = \frac{\int M'y . \frac{ds}{EI}}{\int \frac{y^2}{EI} ds + \int \frac{\cos^2\theta}{EA} . ds} \quad \text{— (1)}$$

At the Crown of arch $\theta = 0^0$

$$\cos\theta = \cos(0) = 1 .$$

$\therefore \frac{A}{\cos\theta} = A_m$ (A_m = Average Cross-Sectional Area).

(θ = Slope of the arch).

$$\int \frac{\cos^2\theta}{EA} . ds = \int \frac{\cos\theta}{EA_m} . ds$$

$$= \int_0^l \frac{1}{EA_m} . ds \quad [\because \cos\theta = 1]$$

$$= \frac{L}{EA_m} .$$

Substitute the Value of above one in Equation (1)

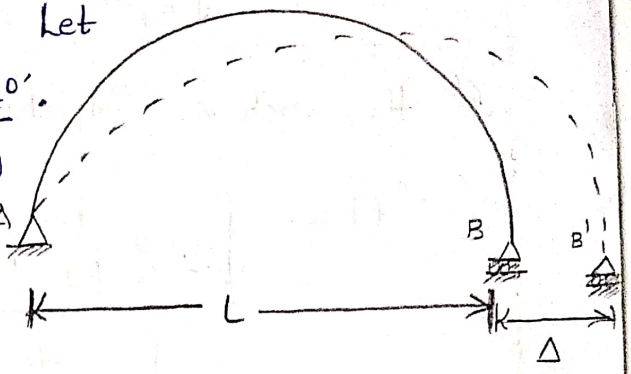
$$H = \frac{\int M'y . \frac{ds}{EI}}{\int \frac{y^2}{EI} ds + \frac{L}{EA_m} + K}$$

K = yielding of Supports

Unit Horizontal force.

Effect of Temperature Changes :-

Consider the arch shown in figure. Let the temperature of arch increase by 't^0'. If the Hinge at ~~beam~~ 'B' is replaced by a roller support, Let End B moves the position to B'.



Then $BB' = L\alpha t$.

Where, L = Span of the arch

α = Coefficient of thermal Expansion

t = Temperature

Let 'H' be the force required to bring back End B' to B, which infers that H is the Horizontal thrust developed in the arch, then $\frac{du}{dH} = L\alpha t$.

$M = M' - Hy$ Because of no loads, there is no reactions ($M' = 0$).

$M = -Hy$.

$$U = \int \frac{M^2}{2EI} \cdot ds$$

$$L\alpha t = \left[\frac{d}{dH}\int \frac{M^2}{2EI} \cdot ds\right] \qquad \frac{dM}{dH} = (-y)$$

$$L\alpha t = \left[\int \frac{2M}{2EI} \cdot \frac{dM}{dH} \cdot ds\right]$$

$$L\alpha t = \int \frac{(-Hy)}{EI} \cdot (-y)\, ds$$

$$L\alpha t = \int \frac{Hy^2}{EI} \cdot ds$$

$$H = \frac{L\alpha t}{\int \frac{y^2}{EI} \cdot ds}$$

If the arch is Subjected to External Loads

$$H = \frac{\int M'y \frac{ds}{EI} + L\alpha t}{\int y^2 \frac{ds}{EI}}$$

In addition to this arch is Subjected to Elastic Shorten of rib yielding of Supports.

$$H = \frac{\int M'y \frac{ds}{EI} + L\alpha t}{\int y^2 \frac{ds}{EI} + \frac{L}{EA_m} + K}$$

$L\alpha t \rightarrow$ temp Changes.

$K \rightarrow$ yielding of Supports

$\frac{L}{EA_m} \rightarrow$ Elastic Shortening of rib.

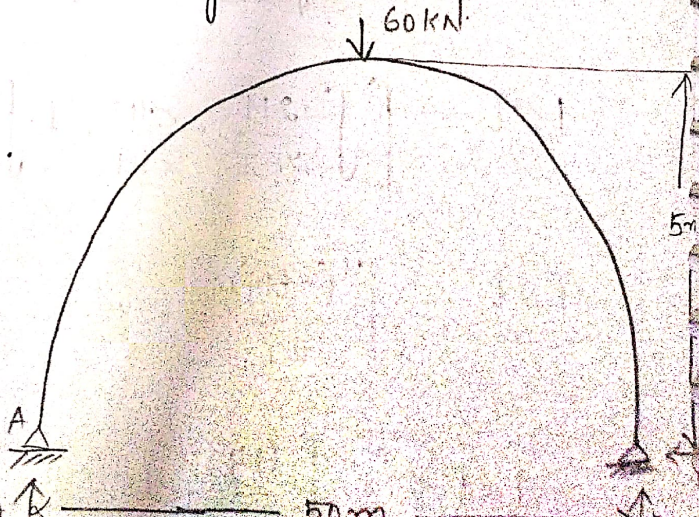
Q:- A Two Hinged Parabolic arch of Span 50 m & rise 5m is Subjected to a Central Concentrated load of 60 kN. It has an Elastic Support which yields by 0.0001 mm/kN and $E = 200\ kN/mm^2$ and $I = 5 \times 10^9\ mm^4$. Average area $A_m = 10,000\ mm^2$, $\alpha = 10 \times 10^{-6}/^oc$ and assuming Secant Variation, Calculate the Horizontal Thrust developed, When the temperature rises by 20^oc.

(i) Neglecting rib Shortening.

(ii) Considering rib Shortening.

Sol:-

$$H = \frac{\int M'y \frac{ds}{EI} + L\alpha t}{\int y^2 \frac{ds}{EI} + \frac{L}{EA_m} + K}$$



Step ①:- Support reactions

$R_A + R_B = 60\ kN$

$\therefore R_A = R_B = 30\ kN$. (Symm Load is acting at Center).

② Beam moment:

$M' = 30x$.

$$\int M'y \cdot \frac{dx}{EI} = 2\int_0^{25} 30x \times \frac{4h x(l-x)}{l^2} \times \frac{dx}{EI}$$

$$= 240 \int_0^{25} \frac{5x^2(50-x)}{50^2} \times \frac{dx}{EI}$$

$$= \frac{12}{25}\int_0^{25} \frac{(50x^2 - x^3)\,dx}{EI} \Rightarrow \frac{12}{25EI}\int_0^{25} (50x^2 - x^3)\,dx$$

$$= \frac{12}{25EI}\left[\frac{50x^3}{3} - \frac{x^4}{4}\right]_0^{25} \Rightarrow \frac{12}{25EI}\left[\frac{50 \times 25^3}{3} - \frac{25^4}{4}\right]$$

$$\frac{78125}{EI}$$

$EI = 200 \times 10^6 \times 5 \times 10^9 \times 10^{-12}$

$= 10^6\ kN \cdot m^2$.

$$\int M'y \cdot \frac{dx}{EI} = \frac{78125}{10^6} = 0.078125.$$

(ii) $$\int y^2 \cdot \frac{dx}{EI} = \int_0^{50} \left[\frac{4h x(l-x)}{x^2}\right]^2 \frac{dx}{EI}$$

$$\Rightarrow \int_0^{50} \left[\frac{4 \times 5 \times x(50-x)}{50^2}\right]^2 \frac{dx}{EI}.$$

$$\Rightarrow \left\{\frac{1}{125EI}\int_0^{50}\left[(50x - x^2)\right]\right\}^2 \frac{dx}{EI}.$$

$$\frac{1}{15625}\int_0^{50}\left[(50^2x^2 + x^4 - 2 \times 50x(x^2)\right] \frac{dx}{EI}$$

$$\frac{1}{15625EI}\left[\frac{50^2\times x^3}{3}+\frac{x^5}{5}-100\times\frac{x^4}{4}\right]_0^{50}$$

$$\Rightarrow \frac{1}{15625EI}\left[\frac{1}{1500}\right]$$

$$\Rightarrow \frac{1}{15625\times10^6}\left(\frac{1}{1500}\right)$$

$$\Rightarrow 6.66\times10^{-4}$$

(iii) $EA_m = 200\times10^6\times10,000\times10^{-6}$

$= 2\times10^6$ KN.

$K = 0.0001$ mm/KN.

$Lat = 50\times10\times10^{-6}\times90$

$= 1/100.$

① Neglecting rib Shortening.

$$H = \frac{\int M'y\frac{dx}{EI}+Lat}{\int y^2\frac{dx}{EI}+K.}$$

$$= \frac{0.078125+1/100}{6.66\times10^{-4}+0.001\times10^3}$$

$= 115.045$

② Considering rib Shortening

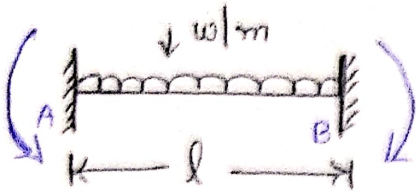
$$H = \frac{\int M'y\frac{dx}{EI}+Lat}{\int y^2\times\frac{dx}{EI}+\frac{L}{EA_m}+K.}$$

$$H = \frac{0.078125 + 1/100}{6.66\times10^{-4} + 0.0001 + \frac{50}{2\times10^{6}}}$$

$$= 111.409$$

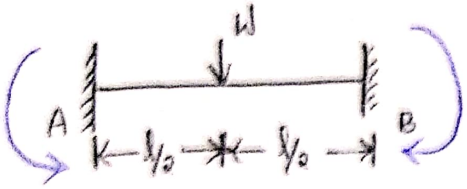
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Fixed End Moments



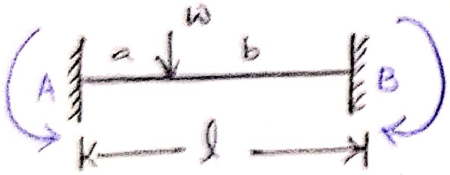
$$M_{FAB} = \frac{-wl^2}{12}$$

$$M_{FBA} = \frac{wl^2}{12}$$

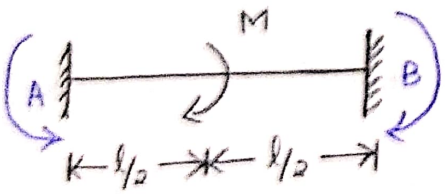


$$M_{FAB} = -wl/8$$

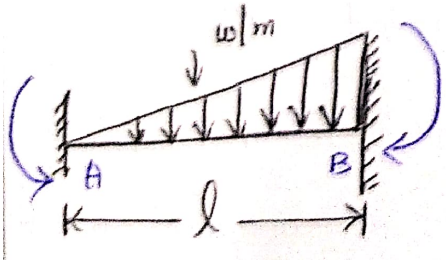
$$M_{FBA} = wl/8$$



$$M_{FAB} = \frac{-wab^2}{l^2}, \quad M_{FBA} = \frac{wa^2b}{l^2}$$

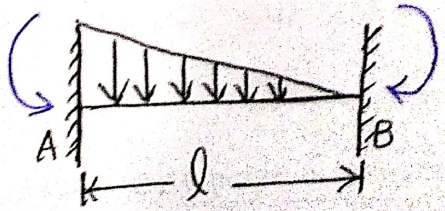


$$M_{FAB} = -M/4 \qquad M_{FBA} = M/4$$



$$M_{FAB} = \frac{-wl^2}{30}$$

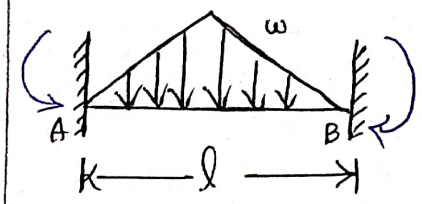
$$M_{FBA} = wl^2/20$$



$$M_{FAB} = \frac{-wl^2}{20}$$

$$M_{FBA} = \frac{wl^2}{30}$$

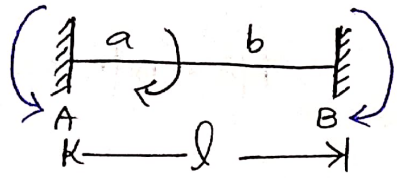
7]



$$M_{FAB} = -\frac{5wl^2}{96}$$

$$M_{FBA} = 5wl^2/96$$

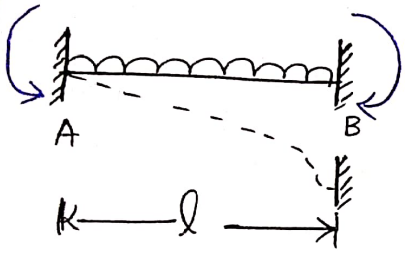
8]



$$M_{FAB} = \frac{Mb}{l}\left(2 - \frac{3b}{l}\right)$$

$$M_{FBA} = \frac{Ma}{l}\left[2 - \frac{3a}{l}\right]$$

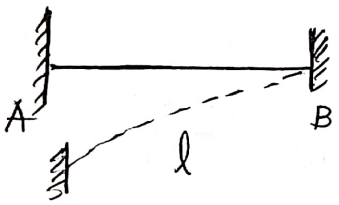
9]



$$M_{FAB} = M_{FBA}$$

$$= -\frac{6EI\delta}{l^2}$$

10]



$$M_{FAB} = M_{FBA} = \frac{6EI\delta}{l^2}.$$

Moment Distribution method :-

It is also called as Hardy Cross method (or) Successive iteration method.

→ It is used for the analysis of indeterminate structure.

In this method first all joints of the structure are assumed to be fixed and fixed end moments are calculated. At this stage moments at a joint are unbalanced, hence release the fixity at a joint and allow the rotation till the moments at a joint are balanced.

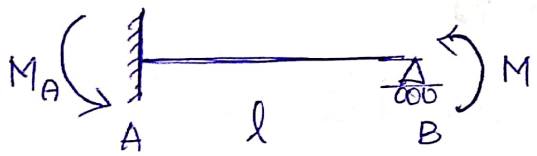
Mathematically it means applying balancing moments which equal and opposite to the unbalanced moment. This balancing moment applied at a joint is appropriately shared by the

members, meeting at the joint, this is called the moment distribution method.

Carry Over moment

When a moment is applied at one end of a member allowing rotation of that end and fixing far end, some moment develops at the far end also this moment is called as carry over moment.

Case 1:- When the far end is fixed.



$$\Delta_1 = \frac{Ml^2}{2EI} \qquad \Delta = \frac{R_B l^3}{3EI}$$

$$\Rightarrow \quad \frac{Ml^2}{2EI} = \frac{R_B l^3}{3EI}$$

$$\Rightarrow \boxed{R_B = \frac{3M}{2l}}$$

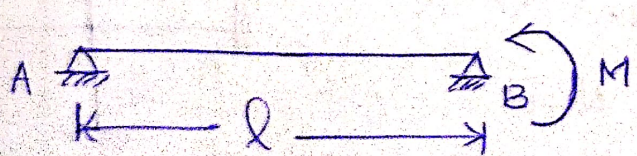
$$\Sigma M_A = 0$$

$$R_B \times l - M - M_A = 0$$

$$\frac{3M}{2l} \times l - M - M_A = 0$$

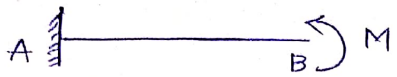
$$\boxed{M_A = M/2}$$

Case 2:- When the far end is Hinged.



$$\boxed{M_A = 0}$$

Case (iii) :- Cantilever Beam



$M_A = -M$

2) Carry Over factor :- If the ratio of Carry over moment to applied moment is called as Carry over factor.

Case (i) :- $COF = \frac{M_A}{M}$

(ii) $COF = \frac{M/2}{M} = 1/2$

(iii)  $COF = 0/M = 0$

iv)  $COF = -M/M = -1$.

Stiffness (k) :-

Moment required to rotate and end by Unit angle (rad ans) when the rotation is Transmitted at that end is called Stiffness of the beam.

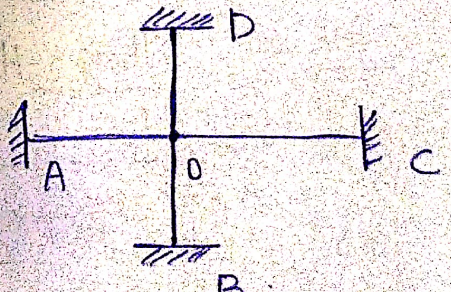
$$K = \frac{M}{\theta_A}$$

(i) When the far end is fixed or Continuous $K = \frac{4EI}{l}$ (or) $\frac{I}{l}$.

(ii) When the far end is hinged $K = \frac{3EI}{l}$ or $\frac{3I}{4l}$

Distribution factor :-

When a moment is applied to a rigid Joint where a No: of members are meeting, applied moment

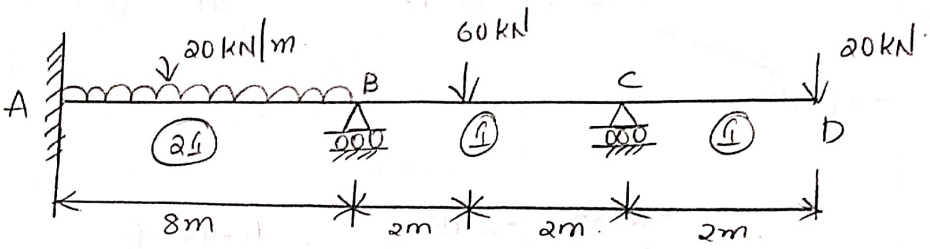


is Shared
the momen
Joint is
D_{OA} =
Q:- Analyse
Sol: Step
M_{FA}
M_{FB}
M_F
M_E
M_I
S

is shared by a members meeting at the joint. The ratio of the moment shared by a member to the applied moment at the joint is called the distribution factor of that member.

$$D_{OA} = \frac{M_{OA}}{M} \quad (or) \quad D_{OA} = \frac{k_{OA}}{K}$$

Q:- Analyse the Beam ABCD shown in figure.



Sol: Step 1: Fixed End Moments :-

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-20\times8^2}{12} = -106.66 \text{ kN.m}$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{20\times8^2}{12} = 106.66 \text{ kN-m}.$$

$$M_{FBC} = \frac{-wl}{8} = \frac{-60\times4}{8} = -30 \text{ kN.m}.$$

$$M_{FCB} = \frac{wl}{8} = \frac{60\times4}{8} = 30 \text{ kN.m}.$$

$$M_{FCD} = -20\times2 = -40 \text{ kN.m}.$$

Step 2: Calculate distribution factors.

Joint	member	Stiffness	Total stiffness	distribution factor
B	BA	$\frac{4EI}{l} = \frac{4E\times2I}{8} = EI$	$\frac{7EI}{4}$	$\frac{EI}{7EI/4} = 0.57$
		$\frac{3EI}{l} = \frac{3EI}{4}$		$\frac{3EI/4}{7EI/4} = 0.43$
	CB	$\frac{4EI}{l} = \frac{4EI}{4} = EI$	EI	1
C	CD	0		0

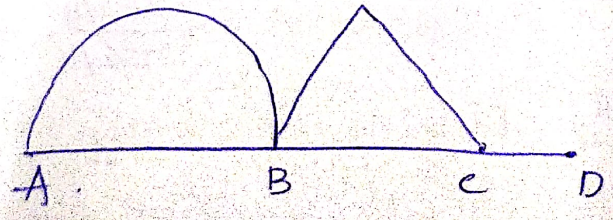
Step 3: Distribution Table

Joints	A	B		C	D
DF		0.57	0.43	1	0
Fixed end moments	-106.67	106.67	-30	30	-40
Balancing moments		-43.7	-32.9	10	0
CoM	-21.85		5	-16.45	0
		-2.85	-2.15	16.45	0
CoM	-1.425		8.225	-1.075	0
Balancing moments		-4.68	-3.53	1.075	0
CoM	-2.34		0.53	-1.76	0
Balancing moments		-0.3	-0.22	1.76	0
CoM	-0.15		0.88	0.1	0
	-132.28	55.14	-55.02	40	-40

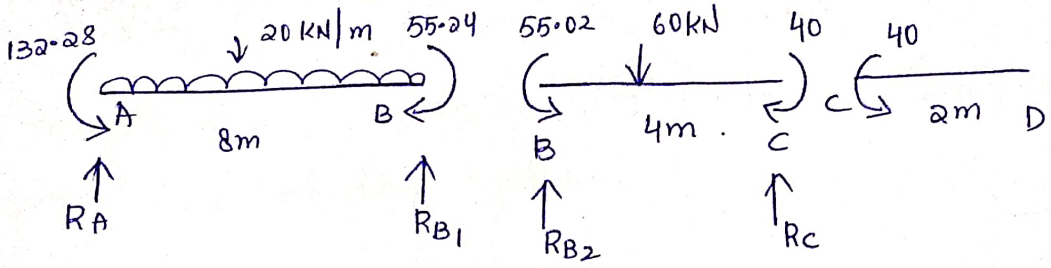
Step 4: Bending moment Diagram.

$$BM\ @B = \frac{wl^2}{8} = \frac{20 \times 8^2}{8} = 160\ kN \cdot m.$$

$$BC = \frac{wl}{4} = \frac{60 \times 4}{4} = 60\ kN \cdot m.$$



Step 5 :- Shear force Diagram



$R_A + R_{B_1} = 20 \times 8$

$R_A + R_{B_1} = 160$ kN.

C.W = A.C.W

$\Sigma M_B = 0$

$R_A \times 8 + 55.14 = 132.28 + 20 \times 8 \times 8/2$

$R_A = 89.64$ kN.

$\Sigma M_C = 0$

$R_B \times 4 + 60 \times 2 + 55.02 = 40$

$R_B = 33.755$ kN

$\Sigma M_B = 0$

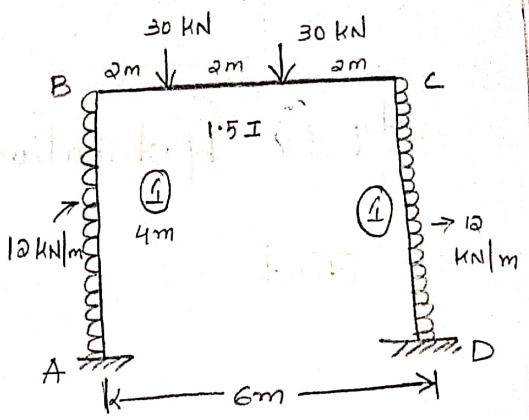
$R_C \times 4 + 55.02 = 40 + 60$

$R_C = 11.245$ kN.

Analysis of frames without Sway:-

Dt:- 18/10/22

Q:- Analysis the Symmetrical Portal frame Shown in figure by the moment distribution method.



Sol:- Step 1:- Calculate fixed end moments.

$$M_{F_{AB}} = \frac{-wl^2}{12} \Rightarrow \frac{-12\times4^2}{12}$$

$$= -16 \text{ kN·m}.$$

$$M_{F_{BA}} = 16 \text{ kN·m}$$

$$M_{F_{BC}} = \frac{-wab^2}{l^2} - \frac{wab^2}{l^2}$$

$$= \frac{-30\times2\times4^2}{6^2} - \frac{30\times4\times2^2}{6^2}$$

$$= -40 \text{ kN·m}.$$

$$M_{F_{CB}} = \frac{wab^2}{l^2} + \frac{wab^2}{l^2}$$

$$= 40 \text{ kN·m}.$$

$$M_{F_{CD}} = \frac{-wl^2}{12} = \frac{-12\times4^2}{12} = -16 \text{ kN·m}.$$

$$M_{F_{DC}} = 16 \text{ kN·m}.$$

Step 2:- Distribution factors.

Joint	member	K	ΣK	D.F = K/ΣK
B	BA	$\frac{4EI}{l} = \frac{4EI}{4} = EI$	2EI	0.5
	BC	$\frac{4EI}{l} = \frac{4E(1.5I)}{6} = EI$		0.5

C	CB	$\frac{4E(1.5I)}{6}$	2EI	0.5
	CD	$\frac{4EI}{4}$		0.5

Step 3:- Distribution table

Joint	A	B		C		D
		0.5	0.5	0.5	0.5	
	−16	16	−40	40	−16	16
		12	12	−12	12	
	6		−6	6		−6
		3	3	−3	3	
	1.5		−1.5	1.5		−1.5
		0.75	0.75	−0.75	−0.75	
final moments	−8.5	31.75	−31.75	31.75	−31.75	8.5

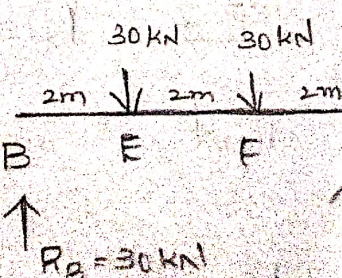
Step 4:- Bending moment Diagram

Span AB = $\frac{wl^2}{8} = \frac{12 \times 4^2}{8}$ = 24 kN.m

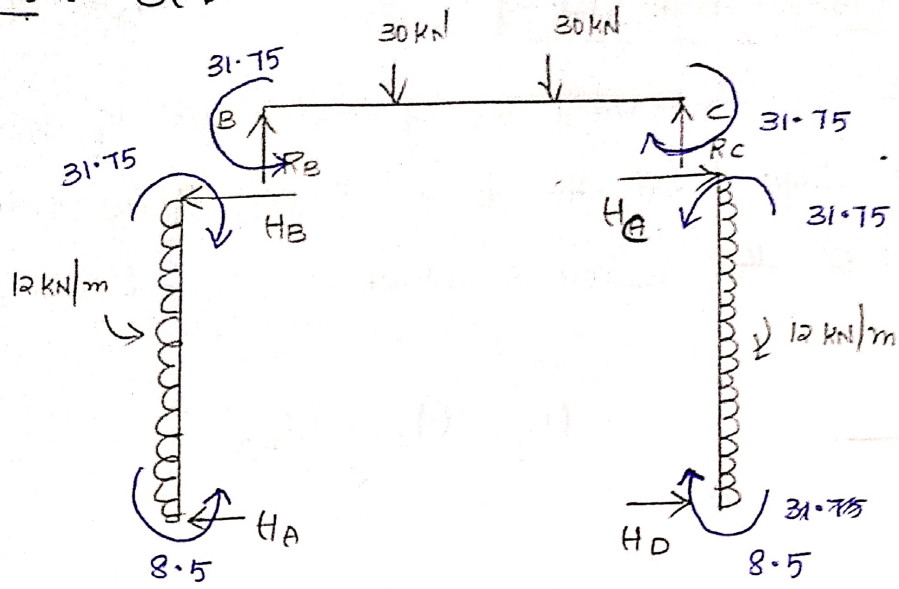
Span BC, $M_E = M_F$ = 60 kN

Span CD = $\frac{wl^2}{8} = \frac{12 \times 4^2}{8}$ = 24 kN.m

R_B & R_C = 30 kN



Step 5 :- SFD



Step 6 :- Final moments

C·ω = A·C·ω

$\Sigma M_B = 0$

$31.75 + H_A \times 4 = 8.5 + 12 \times 4 \times 4/2$

$H_A = 18.18$ kN

$H_B = 29.82$ kN

$\Sigma M_C = 0$

C·ω·M = A·C·ω·M

$31.75 + R_B \times 6 = 31.75 + 30 \times 4 + 30 \times 2$

$R_B = 30$ kN $R_C = 30$ kN

$\Sigma H_B = 0$

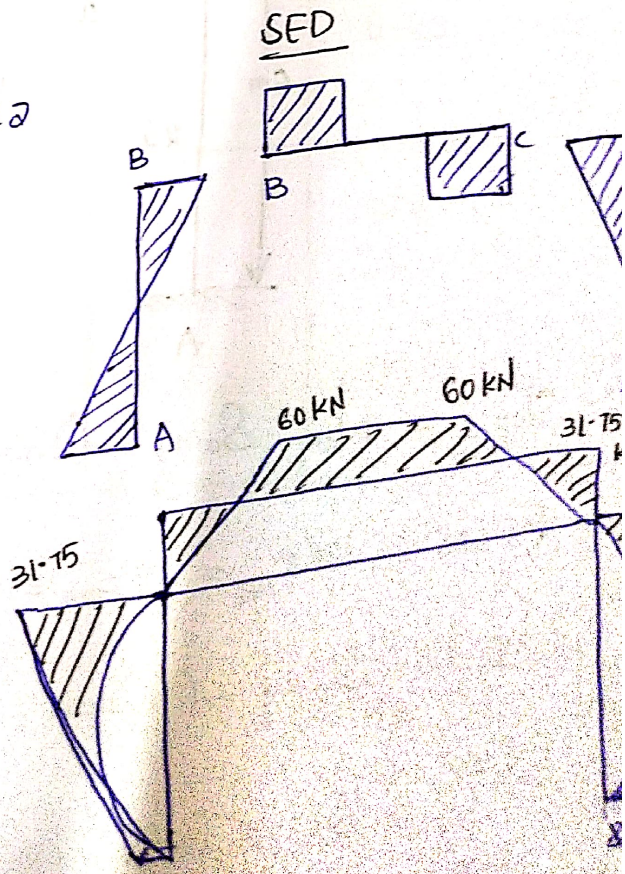
CωM = ACWM

$8.5 + H_C \times 4 = 31.75 + 12 \times 4 \times 4/2$

$H_C = 29.82$ kN

$H_D = 18.18$ kN

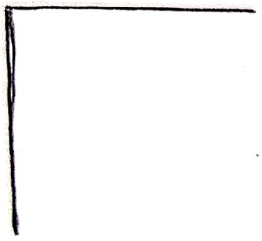
SFD



Dt:- 19/10/22

Analysis of Frames with Sway:-

If there is no Symmetry in geometry (or) in loading & if there is no Support at the beam level, there will be lateral moments of the Column, which is called as Sway.



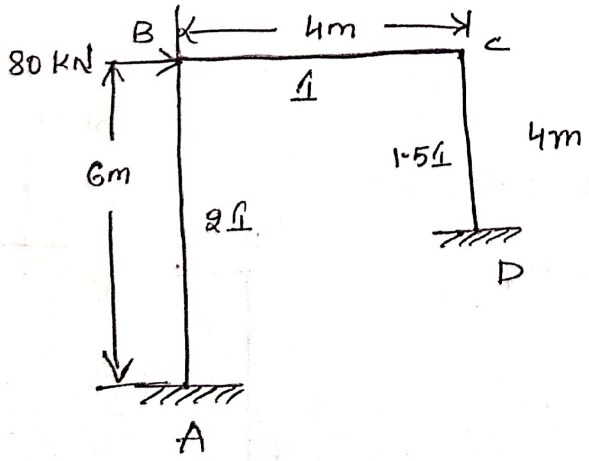
$$H_A = \frac{M_{AB} + M_{BA}}{l_{AB}}$$

$$H_D = \frac{M_{CD} + M_{DC}}{l_{CD}}$$

Sway Correction factor $k = \frac{S}{S'}$

Final Moments = Non Sway moments + [k x Sway moments].

:- Analyse the rigid frame shown in figure by moment distribution method.



Step 1:- Non Sway Analysis

FEM's :- $M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = M_{FCD} = M_{FDC} = 0$

Step 2: Sway Analysis.

Column DC $= \frac{6EIS}{l^2} = \frac{6E(1.5I)S}{4^2}$

$= \frac{9EIS}{16}$

$\frac{\text{FEM Column AB}}{\text{FEM Column DC}} = \frac{EIS/3}{9EIS/16}$

$= \frac{16}{27}$.

FEM's in Column AB = −16 kN·m.

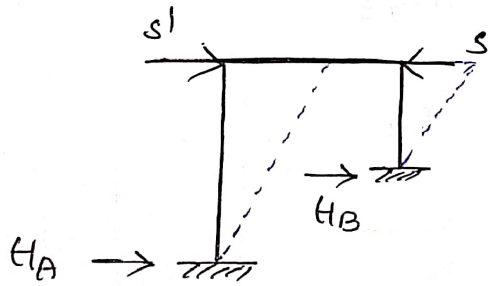
FEM's in Column DC = −27 kN·m.

Step 3: Distribution factor

Joint	members	K	ΣK	D.F = K/ΣK
B	BA	$\frac{4EI}{l} = \frac{4E(2I)}{6} = 4EI/3$	$\frac{7EI}{3}$	0.57
	Bc	$\frac{4EI}{4} = EI$		0.43
C	CB	$\frac{4EI}{4} = EI$	2.5EI	0.4
	CD	$\frac{4EI(1.5I)}{4} = 1.5EI$		0.6

Step 4:- Distribution table

Joints	A	B		C		D
D.F		0.57	0.43	0.4	0.6	
FEM	-16	-16			-27	-27
Bal moments		9.12	6.88	10.8	16.2	
COM	4.56		5.4	3.44		8.1
		-3.078	-2.322	-1.376	-2.064	
	-1.555	0	-0.688	-1.161	0.70	=1.032
		0.392	0.29	0.464	0.696	
final moments	-12.99	-9.5	9.5	12.15	-12.15	-19.93



Sway Correction factor (k) = $\frac{s}{s'}$

$$s' + H_A + H_B = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{l} = \frac{-12.99 - 9.5}{6} = -3.7$$

$$H_B = \frac{M_{BA}^{CD} + M_{BB}^{DA}}{l} = \frac{-12.99 - 19.93}{4} = -8.01$$

$$\delta' - 3.7 - 8.01 = 0$$

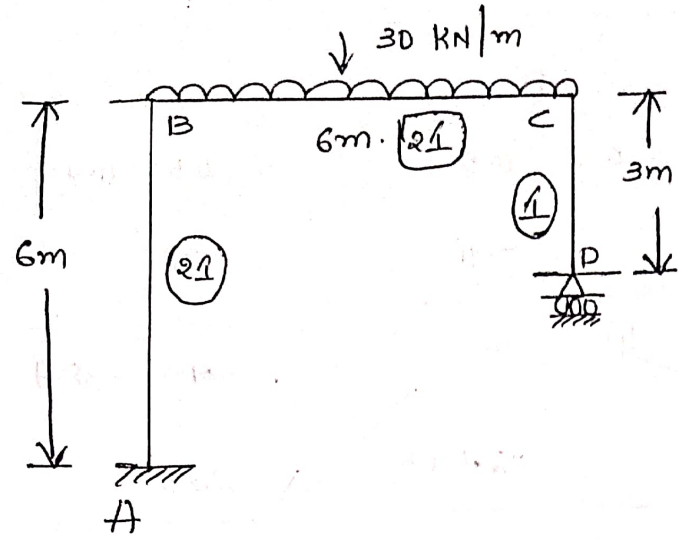
$$\delta' = 11.7$$

$$k = \frac{\delta}{\delta'} = \frac{80}{11.7} = 6.83$$

Step 5: final moments

Sway moments	-12.99	-9.5	9.5	12.15	-12.15	-19.9
Actual Sway moments	-88.72	-64.88	64.88	82.98	-82.98	-135.91
Non Sway Analysis moments	0	0	0	0	0	0
final moments	-88.72	-64.88	64.88	82.98	-82.98	-135.9

Q:- Analyse the frame shown in figure by moment distribution method.



Sol:- Step 1:- Non Sway Analysis.

FEM's:- $M_{FAB} = M_{FBA} = 0$

FEM's:- $M_{FBC} = -\frac{wl^2}{12} = -\frac{30 \times 6^2}{12} = -90 \text{ KN.m}$

$M_{FCB} = \frac{wl^2}{12} = \frac{30 \times 6^2}{12} = 90 \text{ KN.m}$

FEM's:- $M_{FCD} = M_{FDC} = 0$.

Step 2:- Distribution factor:-

Joint	member	K	ΣK	D.F = $K/\Sigma K$.
B	BA	$\frac{4EI}{l} = \frac{4E(2I)}{6}$ $= 4/3\ EI$.	$\frac{8}{3} EI$	0.5
	BC	$\frac{4EI}{l} = \frac{4E(2I)}{6}$ $= 4/3\ EI$		0.5
C	CB	$\frac{4EI}{l} = \frac{4E(2I)}{6}$	$7/3\ EI$	0.57
	CD	$\frac{3EI}{l} = \frac{3EI}{3} = EI$		0.43

Step 3:- Distribution table:-

Joint	A	B		C		D
DF.		0.5	0.5	0.57	0.43	
			-90	90		
		45	45	-51.3	-38.7	
	22.5 ←		-25.65 →	← 22.5		0
		12.82	12.82	-12.82	-9.675	
	6.412 ←		-6.41 →	← 6.41		0
		3.205	3.205	-3.65	-2.75	
	1.60 ←		-1.825 →	← 1.60		0
		0.912	0.912	-0.912	-0.68	
	0.456 ←		→	←		
			-0.456	+0.456		0
	31	62.2	-62.2	52.31	-51.8	0

Step 4: (2) Sway Analysis

FEM's:-

For Column AB $= \frac{-6EI\delta}{l^2} = \frac{-6E(2I)\delta}{6^2} = -\frac{1}{3}EI\delta$

For Column BCD $= \frac{-6EI\delta}{l^2} = \frac{-6E(2I)\delta}{6^2} = -\frac{2}{3}EI\delta$

$$\frac{\text{FEM for AB}}{\text{FEM for DC}} = \frac{+EI\delta/3}{+2EI\delta/3} = \frac{1}{2}$$

$$= \frac{1}{2} \times \frac{20}{40} \frac{50}{50} = \frac{20}{40} \frac{50}{100}$$

FEM's for Column AB = -80 kN·m.

FEM's for Column DC = -100 kN·m.

(ii) Sway factor → next page. (distribution table).

(1)

Sway Correction factor; $(k) = S/S'$

$$S' + H_A + H_D = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{l_{AB}} = \frac{30.51 + 61.92}{6} = 15.40$$

$$H_D = \frac{M_{CD} + M_{DC}}{l_{DC}} = \frac{-51.8 + 0}{3} = -17.26$$

Non sway S' is taken as S'.

$$S' + 15.40 - 17.26 = 0$$

$$S' = -1.86$$

(iii) final moments:- (5)

Sway moments	44.23 ~~30.51~~	−38.36 ~~61.92~~	38.36 −~~61.92~~	59.48 ~~51.83~~	−59.48 −~~51.83~~	0
Actual Sway moments	−44.23 × −0.052 = 2.29	−2.03	2.02	3.14	−3.14	0
non-Sway moments	31	62.2	−62.2	52.31	−51.8	0
F.M.	33.29	60.17	−60.18	55.45	55.4	0

Distribution table :- (3)

Joints	A	B		C		D
DF		0.5	0.5	0.57	0.43	
FEM	-50	-50		-100		-100
		25	25	57	43	
	12.5		28.5	12.5		0
		-14.25	-14.25	-7.125	-5.375	
	-7.125		-3.56	-7.125		0
		1.78	1.78	4.06	3.06	
	0.89		2.03	0.89		0
		-1.01	-1.01	-0.50	-0.38	
	-0.505		-0.25	-0.50		0
		0.125	0.125	0.285	0.215	
	-44.24	-38.36	38.36	59.48	-59.48	-100

(4) **Sway factor :-**

$$K = \frac{S}{S'}$$

$$S' + H_A + H_D = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{l_{AB}} = \frac{-44.24 - 38.36}{6} = -13.78$$

$$H_B = \frac{M_{DC} + M_{CD}}{l_{CD}} = \frac{-59.48 + 0}{3} = -19.91$$

$$S' = 33.75$$

$$\therefore K = \frac{S}{S'} = \frac{-1.86}{33.75} = -0.05$$

Unit - 02

Dt: 21/10/22

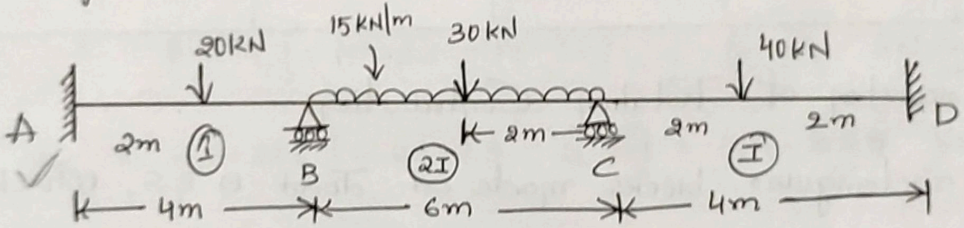
Kani's method

⇒ Rotation factor = $\frac{-1}{2}\left[\frac{K_{AB}}{\Sigma K}\right]$

⇒ Rotation Contribution = R.F [Σ FEM's at that Joint + Rotation Contribution from far End].

⇒ Final moments = ΣFEM's + [2 × near End Contribution) + far End Contribution.

Q:- Analyse the Continuous beam by Kani's method



Sol:- Step ①:- Fixed End moment's

$M_{F_{AB}} = \frac{-\omega l}{8} = \frac{-20\times4}{8} = -10$ kN·m

$M_{F_{BA}} = \frac{\omega l}{8} = 10$ kN·m

$M_{F_{BC}} = \frac{-\omega l^2}{12} - \frac{\omega a b^2}{l^2} \Rightarrow \frac{-15\times6^2}{12} - \frac{30\times4\times2^2}{6^2} \Rightarrow -58.33$ kN·m.

$M_{F_{CB}} = \frac{\omega l^2}{12} + \frac{\omega a^2 b}{l^2} \Rightarrow \frac{15\times6^2}{12} + \frac{30\times4^2\times2}{6^2} \Rightarrow 41.67$ kN·m.

$M_{F_{CD}} = \frac{-\omega l}{8} = \frac{-40\times4}{8} = -20$ kN·m.

$M_{F_{DC}} = \frac{\omega l}{8} = \frac{40\times4}{8} = 20$ kN·m.

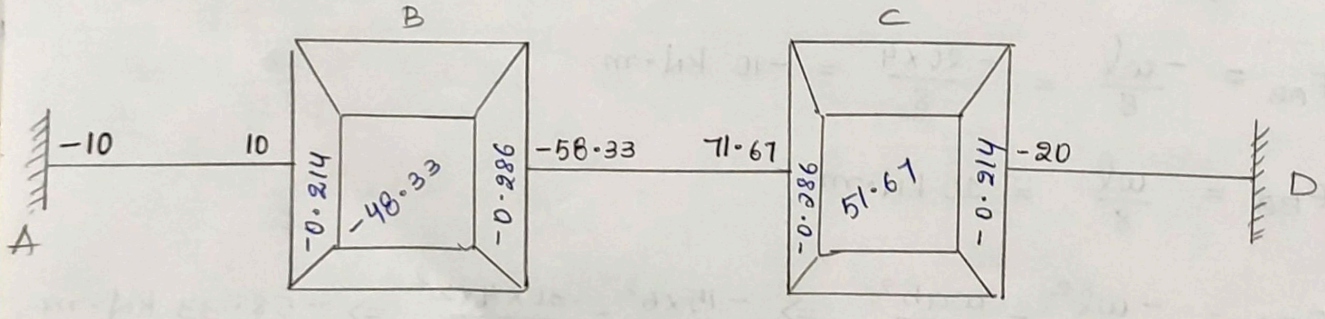
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Step 2 :- Rotation factor

Joints	members	K	ΣK	R.F = $\frac{-1}{2}\left[\frac{K}{\Sigma K}\right]$
B	BA	$\frac{4EI}{l} = \frac{4EI}{4} = EI$	$7/3\,EI$	−0.214
	BC	$\frac{4EI}{l} = \frac{4(2I)E}{6} = 4/3\,EI$		−0.286
C	CB	$\frac{4EI}{l} = \frac{4E(2I)}{6} = 4/3\,EI$	$7/3\,EI$	−0.286
	CD	$\frac{4EI}{l} = \frac{4EI}{4} = EI$		−0.214

Step ③ :- Calculation of Rotation Contributions :-

Double rectangular blocks made at Joint B & C, which rotate during deformation.



10.34	13.8	−18.72	−14.01
14.34	19.17	−20.26	−15.15
14.67	19.61	−20.38	−15.25
14.70	19.65	−20.39	−15.26
14.70	19.65	−20.39	−15.26

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Joint B:

RC M'_{BA} = RF [ΣFEM's @ Joint B + R.C from far end]

= −0.214 [−48.33 + 0] = 10.34 kN·m

RC M'_{BC} = −0.286 [−48.33 + 0] = 13.8 kN·m

Joint C:

RC M'_{CB} = −0.286 [51.67 + 13.81] = −18.72 kN·m

M'_{CD} = −0.214 [51.67 + 13.81] = −14.01 kN·m

(ii) Joint B:

RC M'_{BA} = RF [ΣFEM's @ Joint B + RC from far end]

= −0.214 [−48.33 − 18.72] = 14.34 kN·m

RC M'_{BC} = −0.286 [−48.33 − 18.72] = 19.17 kN·m

Joint C

RC M'_{CB} = −0.286 [51.67 + 19.17] = −20.26 kN·m

M'_{CD} = −0.214 [51.67 + 19.17] = −15.15 kN·m

Cycle ③ :- Joint B

RC M'_{BA} = −0.214 [−48.33 − 20.26] = 14.67 kN·m

M'_{BC} = −0.286 [−48.33 − 20.26] = 19.616 kN·m

Joint C

RC M'_{CB} = −0.286 [51.67 + 19.61] = −20.38 kN·m

M'_{CD} = −0.214 [51.67 + 19.61] = −15.25 kN·m

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Cycle ④ :- Joint B:-

Rc $M'_{BA} = -0.214[-48.33 - 20.38] = 14.70$ kN·m

$M'_{BC} = -0.286[-48.33 - 20.38] = 19.65$ kN·m

Joint C:-

Rc $M'_{CB} = -0.286[51.67 + 19.65] = -20.39$

$M'_{BC} = -0.214[51.67 + 19.65] = -15.26$.

Cycle ⑤ :- Joint B:

Rc $M'_{BA} = -0.214[-48.33 - 20.39] = 14.70$

Rc $M'_{BC} = -0.286[-48.33 - 20.39] = 19.65$

Joint C:-

Rc $M'_{CB} = -0.286[51.67 + 19.65] = -20.39$

Rc $M'_{BC} = -0.214[51.67 + 19.65] = -15.26$

~~Step ⑤~~ :- BMD. Step ④ :-

~~Free Bending~~.

Final moments = ΣFEM's + (2 near end Contributions + far end Contributions).

	AB	BA	BC	CB	CD	DC
FEM's	-10	10	-58.33	71.67	-20	20.
2 × near end Contributions	2×0 = 0	2×14.70 = 29.4	2×19.65 = 39.3	2×-20.39 = -40.78	2×-15.26 -30.52	2×0 = 0.
far end Contributions	14.70	0	-20.39	19.65	0	-15.26.
final moments.	4.7	39.4	-39.4	50.54	-50.52	4.74

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Step ⑤ :- BMD (Free Bending moments).

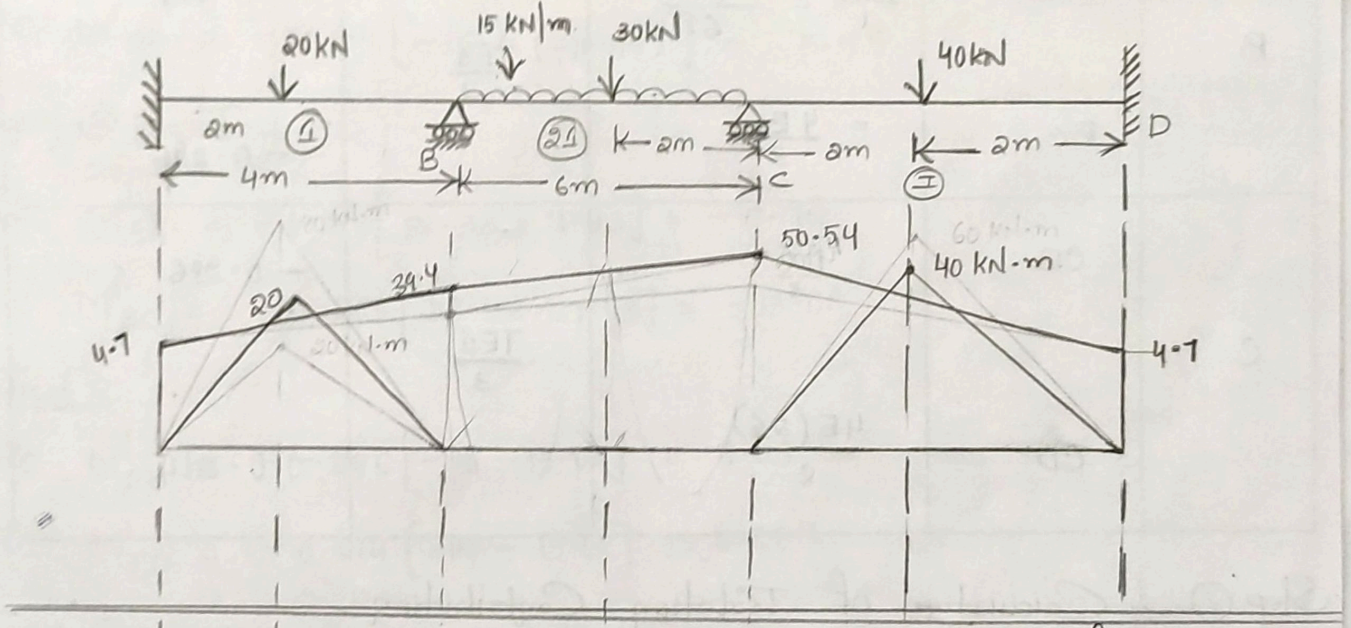
Span AB = $\frac{wl}{4} = \frac{20 \times 4}{4}$ = 20 kN·m

Span BC = $\frac{wl^2}{8} = \frac{15 \times 6^2}{8}$ = 67.5 kN·m (udl)

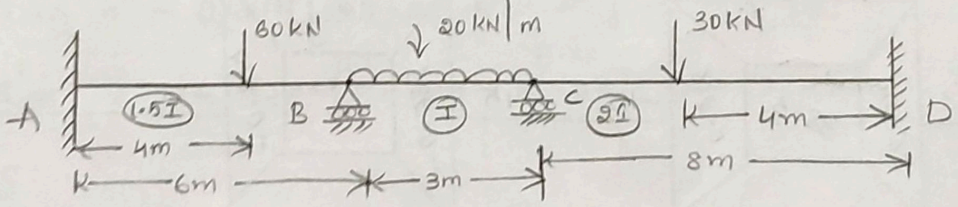
Span Bc = $\frac{Wab}{l} = \frac{30 \times 4 \times 2}{6}$ = 40 kN·m (Point load).

Span Bc = 67.5 + 40 = 107.5 kN·m.

Span CD = $\frac{wl}{4} = \frac{40 \times 4}{4}$ = 40 kN·m.



Q:- Analyse the Continuous beam by Kani's method.



Step ① :- Fixed End moments.

$M'_{FAB} = \frac{-wab^2}{l^2} = \frac{-60 \times 4 \times 2^2}{6^2}$ = -26.67 kN·m.

$M_{FBA} = \frac{+wa^2b}{l^2} = \frac{60 \times 4^2 \times 2}{6^2}$ = 53.3 kN·m.

$M_{FBC} = \frac{-wl^2}{12} = \frac{-20 \times 3^2}{12}$ = -15 kN·m.

$M_{FCB} = wl^2/12$ = 15 kN·m.

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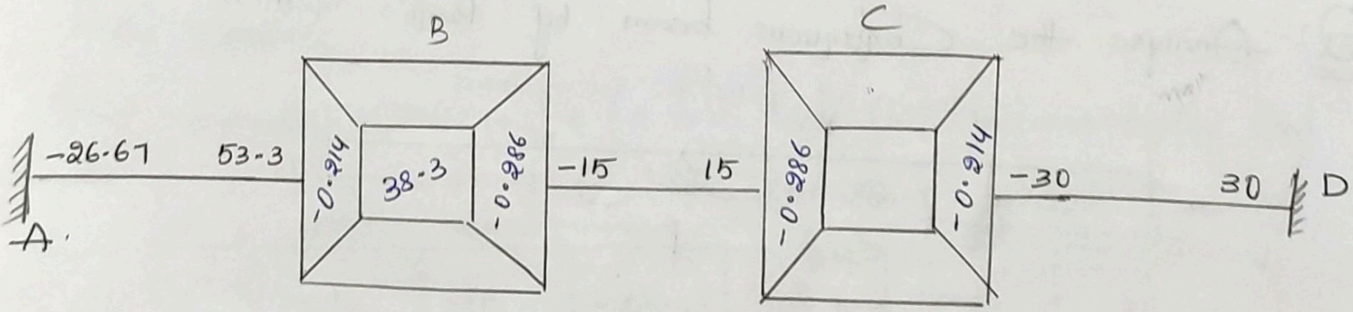
$M_{FCD} = \frac{-\omega l}{8} = \frac{-30 \times 8}{8} = -30$ kN-m.

$M_{FDC} = \omega l/8 = 30$ kN-m.

Step ② :- Rotation factor :-

Joints	members	k.	Σk	R.F $= \frac{-1}{2}\left[\frac{k}{\Sigma k}\right]$
B	BA	$\frac{4EI}{l} = \frac{4E(1.5I)}{6}$	$\frac{7EI}{3}$	-0.214
	BC	$= \frac{4EI}{3}$		-0.286
C	CB	$\frac{4EI}{3}$	$\frac{7EI}{3}$	-0.286
	CD	$\frac{4E(2I)}{8}$		-0.214

Step ③ :- Calculation of Rotation Contribution :-



-8.196	-10.95	7.42	5.55
-9.79	-13.08	8.03	6.009
-9.92	-13.25	8.079	6.045
-9.92	-13.25	8.07	6.04

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Cycle ①: Joint B

Rotation Contribution = R·F [ΣFEM @ the Joint B + RC from far End]

$M'_{BA} = -0.214[38.3 + 0] = -8.196$

$M'_{BC} = -0.286[38.3 + 0] = -10.95$

Joint C:-

RC $M'_{CB} = -0.286[-15 + (-10.95)] = 7.42$

RC $M'_{CD} = -0.214[-15 + (-10.95)] = 5.55$

Cycle ②:- Joint B:

RC $M'_{BA} = -0.214[38.33 + 7.43] = -9.79$

RC $M'_{BC} = -0.286[38.33 + 7.43] = -13.08$

Joint C

RC $M'_{CB} = -0.286[-15 - 13.08] = 8.03$

RC $M'_{CD} = -0.214[-15 - 13.08] = 6.009$

Cycle ③: Joint B:

RC $M'_{BA} = -0.214[38.33 + 8.03] = -9.92$

RC $M'_{BC} = -0.286[38.33 + 8.03] = -13.25$

Joint C:

RC $M'_{CB} = -0.286[-15 - 13.25] = 8.079$

RC $M'_{CD} = -0.214[-15 - 13.25] = 6.045$

Cycle ④: Joint B:

RC $M'_{BA} = -0.214[38.33 + 8.07] = -9.92$

$M'_{BC} = -0.286[38.33 + 8.07] = -13.25$

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Joint C:

Rc $M'_{CB} = -0.286[-15 - 13.25] = 8.07$

Rc $M'_{CB} = -0.214[-15 - 13.25] = 6.04$

Step ④:

Final moments = ΣFEM's + (2 near End Contribution + far End Contributions).

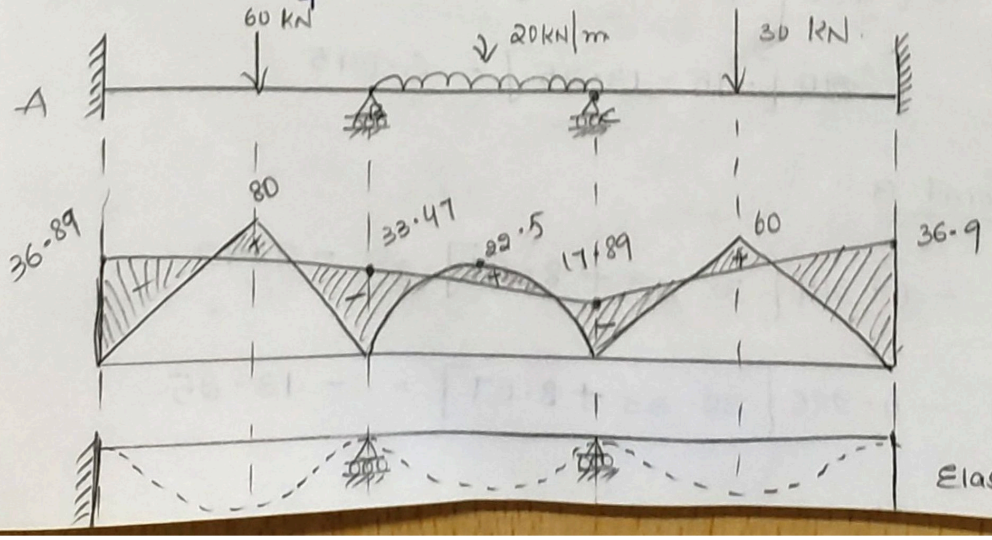
FEM's.	-26.67	53.33	-15	15	-30	30
2 x near End Contributions	2x0 = 0	2x-9.93 = -19.86	2x-13.77 = -26.54	2x 8.08 = 16.16	2x 6.04 = 12.08	2x0 = 0
far End Contribution	-9.93	0	8.08	-13.27	0	6.04
final moments	-36.59	33.47	-33.47	17.89	-17.9	36.04

Step ⑤: BMD (free Bending moments).

Span AB = $\frac{wab}{l} = \frac{60 \times 4 \times 2}{6} = 80$ kN.m.

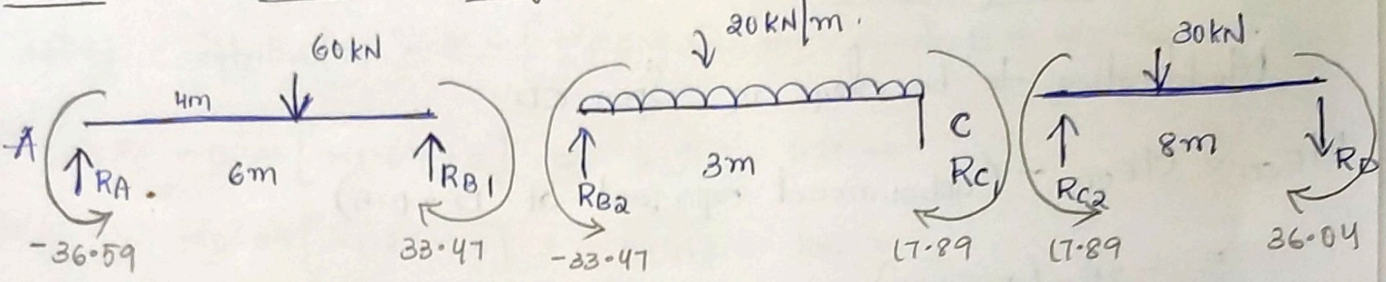
Span BC = $\frac{wl^2}{8} = \frac{20 \times 3^2}{8} = 22.5$ kN.m.

Span CD = $\frac{wl}{4} = \frac{30 \times 8}{4} = 60$ kN.m



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Step ⑥ :- Shear force diagram :-



$R_A \times 6 + 33.47 = (60 \times 2) + (36.59)$

$R_A \times 6 = 123.12$ kN

$R_A = 20.52$ kN

$R_A + R_{B1} = 60$ kN

$R_{B1} = 60 - 20.52$

$R_{B1} = 39.48$ kN

$R_{B2} + R_{C1} = 20 \times 3$

$R_{B2} \times 3 + 17.89 = 33.47 + \left(20 \times 3 \times \frac{3}{2}\right)$

$R_{B2} = 35.193$ kN

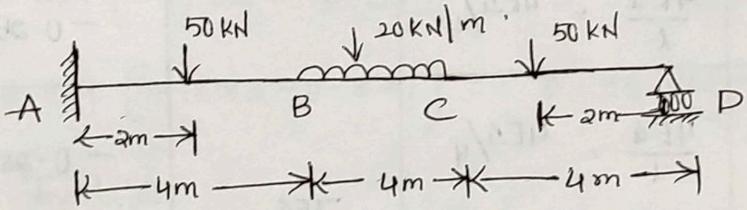
$R_{C1} = 24.80$ kN

$R_{C2} \times 8 + 36.04 = 30 \times 4 + 17.89$

$R_{C2} = 12.73$ kN

$R_D = 17.27$ kN

Q:- Analyse the Continuous Beam by Kani's method.



Sol:- Step 1 :- Calculate the fixed end Moments.

① $M_{FAB} = \frac{-wl}{8} = \frac{-50 \times 4}{8} = -25$ kN.m.

② $M_{FBA} = wl/8 = 25$ kN.m.

③ $M_{FBC} = \frac{-wl^2}{12} = \frac{-20 \times 4^2}{12} = -26.67$ kN.m.

④ $M_{FCB} = +26.67$ kN.m.

⑤ $M_{FCD} = \frac{-wl}{8} = \frac{-50 \times 4}{8} = -25$ kN.m.

⑥ $M_{FDC} = \frac{wl}{8} = 25$ kN.m.

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Step ② :- Rotation factor :

Modification to be done in Span. CD.

$M_{FCD} = M_{FCD} - (\text{unbalanced moment at 'D'} \times 0.5)$

$= -25 - (0.5 \times 25)$

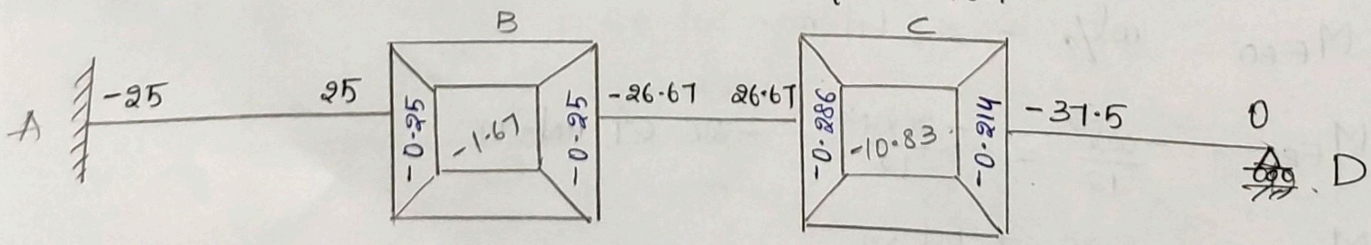
$= -37.5$ kN·m.

$M_{FDC} = 0$. ↓

Rotation factor :

Joint	member	K	Σk	R.F = $-\frac{1}{2} \times \frac{k}{\Sigma k}$
B	BA	$\frac{4EI}{l} = 4EI/4$	$2EI$	-0.25
	BC	$\frac{4EI}{l} = 4EI/4$		-0.25
C	CB	$\frac{4EI}{l} = 4EI/4$	$\frac{7EI}{4}$	-0.286
	CD	$\frac{3EI}{l} = \frac{3EI}{4}$		-0.214

Step ③ Calculation of Rotation Contribution.



0.41	0.41	2.98	2.22
-0.327	-0.327	3.19	2.38
-0.38	-0.38	3.20	2.39
-0.38	-0.38	3.20	2.39

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Cycle ① :- Joint B :-

Rotation Contribution = D.F [ΣFEM @ the Joint B + FC from far End].

$M'_{BA} = -0.25[-1.67+0] = 0.417$ KN·m

$M'_{BC} = -0.25[-1.67+0] = 0.417$ KN·m.

Joint C :-

$M'_{CB} = -0.286[-10.83+0.41] = 2.98$ KN·m

$M'_{BCD} = -0.214[-10.83+0.41] = 2.22$ KN·m

Cycle ② :- Joint B :-

$M'_{BA} = -0.25[-1.67+2.98] = -0.327$ KN·m

$M'_{BC} = -0.25[-1.67+2.98] = -0.327$ KN·m.

C Joint :-

$M'_{CB} = -0.286[-10.83-0.327] = 3.19$ KN·m.

$M'_{CD} = -0.214[-10.83-0.327] = 2.38$ KN·m.

Cycle ③ :- Joint B

$M'_{BA} = -0.25[-1.67+3.19] = -0.38$ KN·m.

$M'_{BC} = -0.25[-1.67+3.19] = -0.38$ KN·m.

Joint C :-

$M'_{CB} = -0.286[-10.83-0.38] = 3.206$ KN·m.

$M'_{CD} = -0.214[-10.83-0.327] = 2.398$ KN·m.

Cycle ④ :- Joint B :-

$M'_{BA} = -0.25[-1.67+3.20] = -0.38$ KN·m

$M'_{BC} = -0.25[-1.67+3.20] = -0.38$ KN·m.

Joint C :- $M'_{CB} = -0.286[-10.83-0.38] = 3.206$ &

$M'_{CD} = 2.398$ KN·m.

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Step ④ :-

Final Moments = ΣFEM's + (2 near End Contribution + far End Contribution).

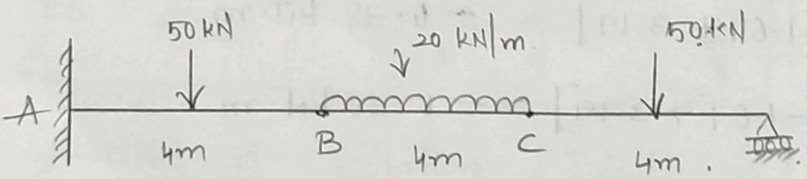
	A	B		C		D
FEM's	-25	25	-26.67	26.67	-25 (37.5)	25 (0)
2× near End Contribution	2×0 = 0	2×-0.38 -0.76	2×-0.38 -0.76	2×3.20 = 6.4	2×2.4 = 4.8	0
far End Contribution	-0.38	0	3.2	-0.38	0	0
final moments.	-25.38	24.24	-24.23	32.69	-32.70	0

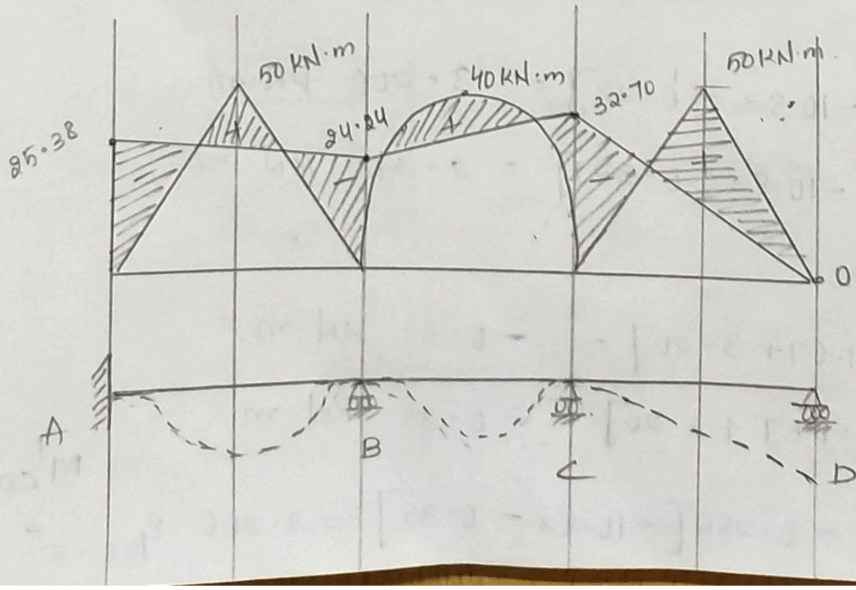
Step ⑤ BMD (free Bending moments) :-

Span AB = $\frac{wl}{4} = \frac{50 \times 4}{4} = 50$ kN.m.

Span BC = $\frac{wl^2}{8} = \frac{20 \times 4^2}{8} = 40$ kN.m.

Span CD = $\frac{wl}{4} = \frac{50 \times 4}{4} = 50$ kN.m.

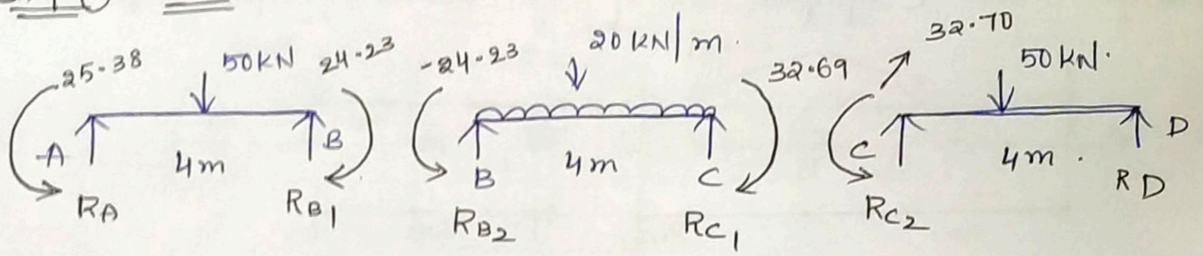




Elastic Curve

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Step ⑥ SFD :-



$\Sigma M_B = 0$

C.W = A.C.W

$R_A \times 4 + 24.23 = (50 \times 2) + 25.38$

$R_A = 26.28$ kN

$R_A + R_{B_1} = 50$

$R_{B_1} = 50 - 26.28 = 23.72$ kN.

$R_{B_1} = 23.72$ kN

$R_{C_2} \times 4 = 50 + 32.70$

$R_{C_2} = 20.675$ kN

$R_{B_2} \times 4 + 32.69 = (20 \times 4 \times \frac{4}{2} + 24.23)$

$R_{B_2} = 37.885$ kN

$R_{C_1} = (20 \times 4) - 37.885$

$R_{C_1} = 42.115$ kN

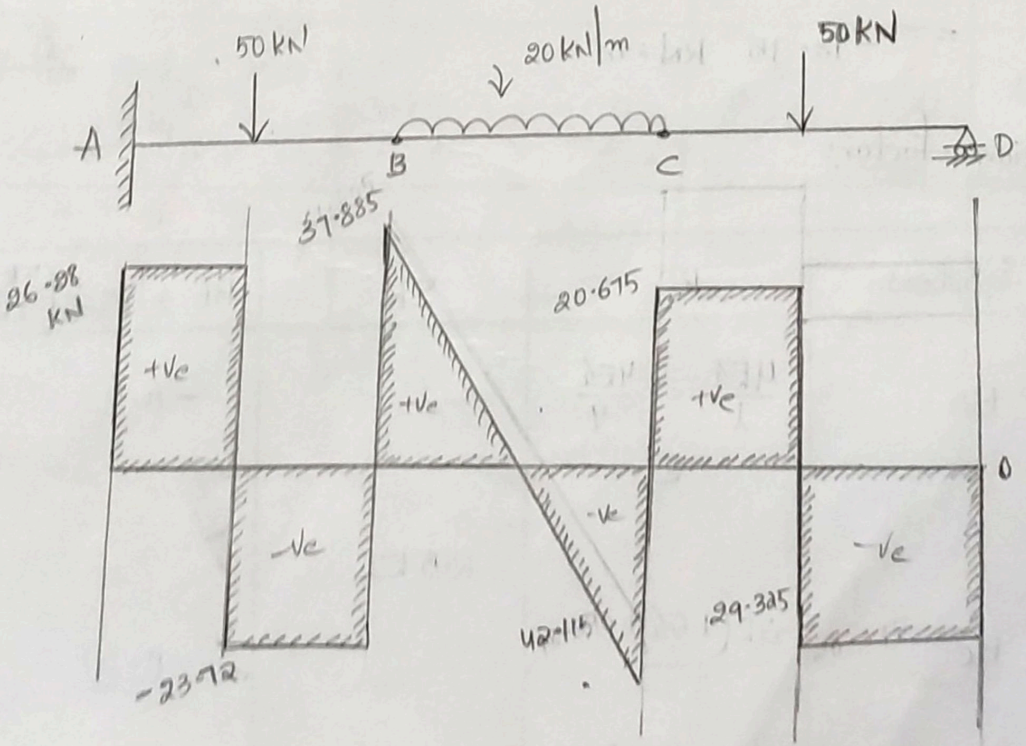
$R_{C_2} + R_D = 50$ kN

$\Rightarrow R_{C_2} \times 4 = 50$

$R_{C_2} = 12.5$ kN

$R_D = 50 - 20.675$

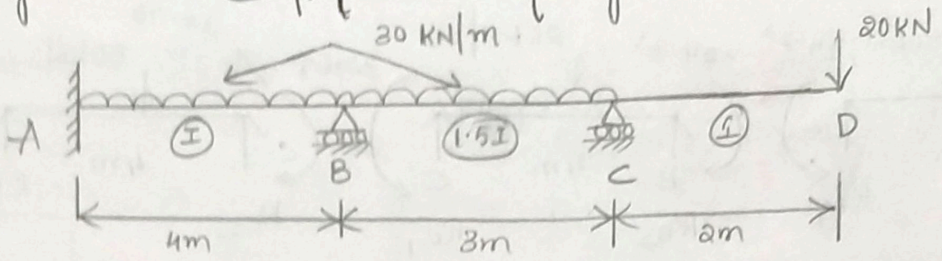
$R_D = 29.325$ kN



37.885
22.115
20.675
42.115

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Q:- Analyse the Continuous beam by Kanni's method.



Sol:- Step ① :- Fixed End moments

$$M_{FAB} = \frac{-wl^2}{12} = -\frac{30\times4^2}{12} = -40 \text{ kN-m}.$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{30\times4^2}{12} = 40 \text{ kN-m}$$

$$M_{FBC} = \frac{-wl^2}{12} = -\frac{30\times3^2}{12} = -22.5 \text{ kN-m}$$

$$M_{FCB} = 22.5 \text{ kN-m}$$

$$M_{FCD} = -20\times2 = -40 \text{ kN-m}.$$

⇒ Modification to take Care of rotation of Support C.

Modified $M_{FBC} = M_{FBC} = -(\text{unbalanced moment at C} \times 0.5)$

$$M_{FBC} = -22.5 - (0.5\times(22.5-40))$$

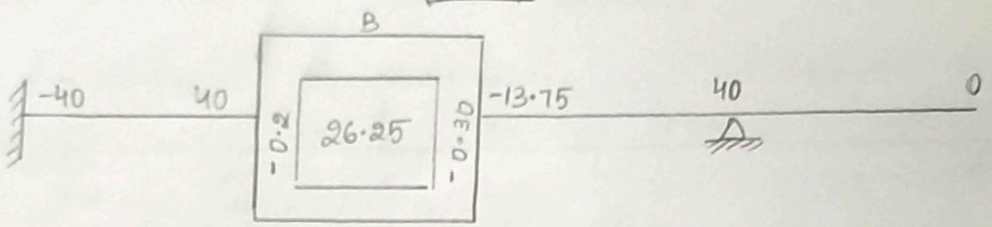
$$= -13.75 \text{ kN-m}.$$

Step ② :- Rotation factor:

Joint	members	K	ΣK	RF = $-\frac{1}{2}\left[\frac{K}{\Sigma K}\right]$
B	BA	$\frac{4EI}{4} = \frac{4EI}{4}$ = EI	2.5 EI	-0.2
	BC	$\frac{3E(1.5I)}{3}$ = 1.5EI		-0.3

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Step ③: Calculation of Rotation Contribution:



Joint B:

-5.25 -7.875

Rotation Contribution = RF [ΣFEM @ the Joint B + RC from far End].

$M'_{BA} = -0.2[26.25+0] = -5.25$

$M'_{BC} = -0.3[26.25+0] = -7.875$

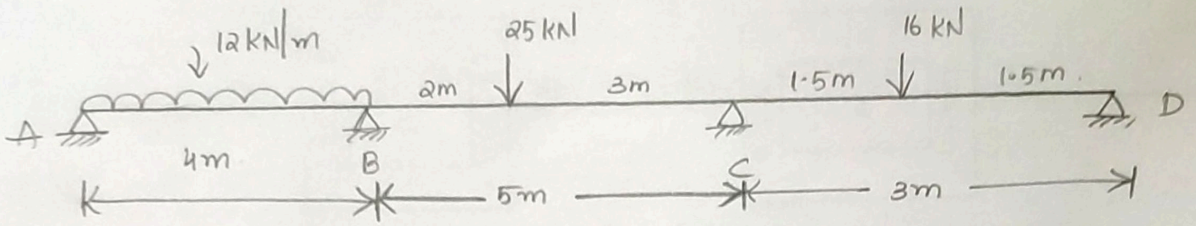
Step ④: final moments.

Final moments = ΣFEM's + (2 near End Contribution + far End Contributions).

	A		B		C	D
FEM's	-40	40	-13.75	40	-40	0
2x near End Contribution	2x0 = 0	2(-5.25) =	2(-7.875) =	0	0	0
far End Contribution	-5.25	0	0	0	0	0
final moments	-45.5	29.5	-29.5	40	-40	0

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Q:- Analyse this beam using Kani's method:



Step ①: Fixed end moments:-

$M_{FAB} = -\frac{wl^2}{12} = -\frac{12\times4^2}{12} = -16$ kN·m.

$M_{FBA} = 16$ kN·m

$M_{FBC} = -\frac{wab^2}{L^2} = -\frac{25\times2\times3^2}{5^2\times12} = -18$ kN·m.

$M_{FCB} = \frac{wa^2b}{l^2} = \frac{25\times2^2\times3}{5^2} = 12$ kN·m

$M_{FCD} = -\frac{wl}{8} = -\frac{16\times3}{8} = -6$ kN·m

$M_{FDC} = wl/8 = 16\times3/8 = 6$ kN·m.

Modification factor for rotation of A & D

$M_{FAB} = 0$

$M_{FBA} = 16 - 0.5(-16) = 24$ kN·m

$M_{FCD} = -6 - 0.5(12-6) = -9$ kN·m

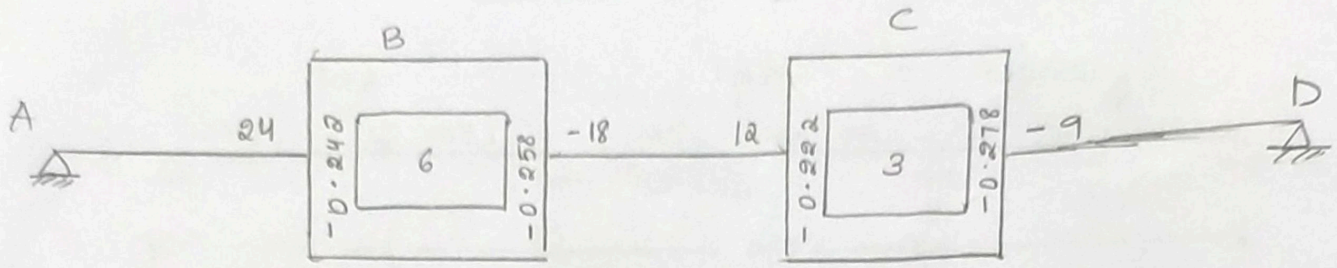
$M_{FDC} = 0$

Step ②: Rotation factor:-

Joint	members	K	ΣK	RF $\left[\frac{-1}{2}\right]\left[\frac{K}{\Sigma K}\right]$
B	BA	$\frac{3EI}{4} = 0.75EI$	1.55 EI	-0.242
	BC	$\frac{4EI}{5} = 0.8EI$		-0.258
C	CB	$\frac{4EI}{5} = 0.8EI$	1.8 EI	-0.22
	CD	$\frac{3EI}{3} = EI$		-0.278

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Step ③ Calculation of Rotation Contribution:



-1.452	-1.548	-0.322	-0.403
-1.374	-1.464	-0.340	-0.427
-1.369	-1.460	-0.341	-0.428
-1.369	-1.460	-0.341	-0.428

Rotation Contribution = [Σ Rotation Contribution at far End + ΣFEM] × RF

Joint B: Cycle ① :-

$M'_{BA} = -0.242[6+0] = -1.452$

$M'_{Bc} = -0.258[6+0] = -1.548$

Joint c :-

$M'_{CB} = -0.222[3-1.548] = -0.322$

$M'_{CD} = -0.278[3-1.548] = -0.403$

Cycle ② :- Joint B

$M'_{BA} = -0.242[6-0.322] = -1.374$

$M'_{Bc} = -0.258[6-0.322] = -1.464$

Joint C :-

$M'_{CB} = -0.222[3-1.464] = -0.340$

$M'_{CD} = -0.278[3-1.464] = -0.427$

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Joint B:- Cycle ③:

$M'_{BA} = -0.242[6 - 0.340] = -1.369$

$M'_{BC} = -0.258[6 - 0.340] = -1.460$

Joint C:

$M'_{CB} = -0.222[3 - 1.460] = -0.341$

$M'_{CD} = -0.278[3 - 1.460] = -0.428$

Cycle ④:- Joint B:

$M'_{BA} = -0.242[6 - 0.341] = -1.369$

$M'_{BC} = -0.258[6 - 0.341] = -1.460$

Joint C:

$M'_{CB} = -0.222[3 - 1.460] = -0.341$

$M'_{CD} = -0.278[3 - 1.460] = -0.428$

Step ⑤:- final Moments

Final moments = ΣFEM's + (2 near End Contribution + far End Contributions).

	A	B		C	D
FEM	0	24	-18	12	-9	0
2x near End Contributions	2x0 = 0	2x(-1.369) = -2.738 -0.342	2x(-1.46) -2.92	2x-0.341 -0.682	2x-0.428 -0.856	0 0
Far End Contribution	0	0	-0.341	-1.460	0	0
Final moments	0	21.26	-21.26	9.858	-9.856	0

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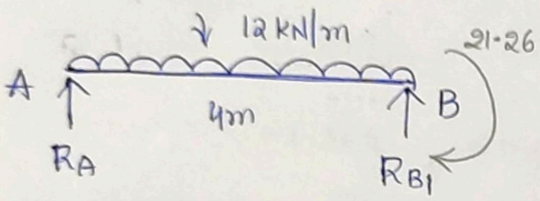
Step ⑤ : BMD (free Bending moments).

Span AB : $\frac{wl^2}{8} = \frac{12 \times 4^2}{8} = 24$ kN·m.

Span BC = $\frac{wab}{l} = \frac{25 \times 2 \times 3}{5} = 30$ kN·m.

Span CD = $\frac{wl}{4} = \frac{16 \times 3}{4} = 12$ kN·m.

Step ⑥ : Shear force



$R_A + R_{B1} = 12 \times 4$

CW = ~~ACW~~

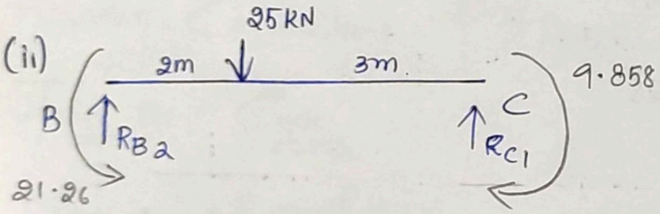
$R_A \times 4 + 21 \cdot 26 = 12 \times 4 \times \frac{4}{2}$

$R_A = 18 \cdot 68$ kN

$\therefore R_{B1} = 48 - 18 \cdot 68$

$R_{B1} = 29 \cdot 32$ kN

(ii)



$R_{B2} + R_{C1} = 25$

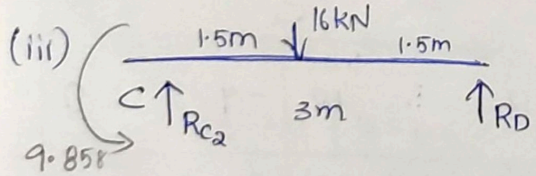
$R_{B2} \times 5 + 9 \cdot 858 = 25 \times 3 + 21 \cdot 26$

$R_{B2} = 17 \cdot 280$ kN

$R_{C1} = 25 - 17 \cdot 280$

$R_{C1} = 7 \cdot 72$ kN

(iii)



$R_{C2} + R_D = 16$

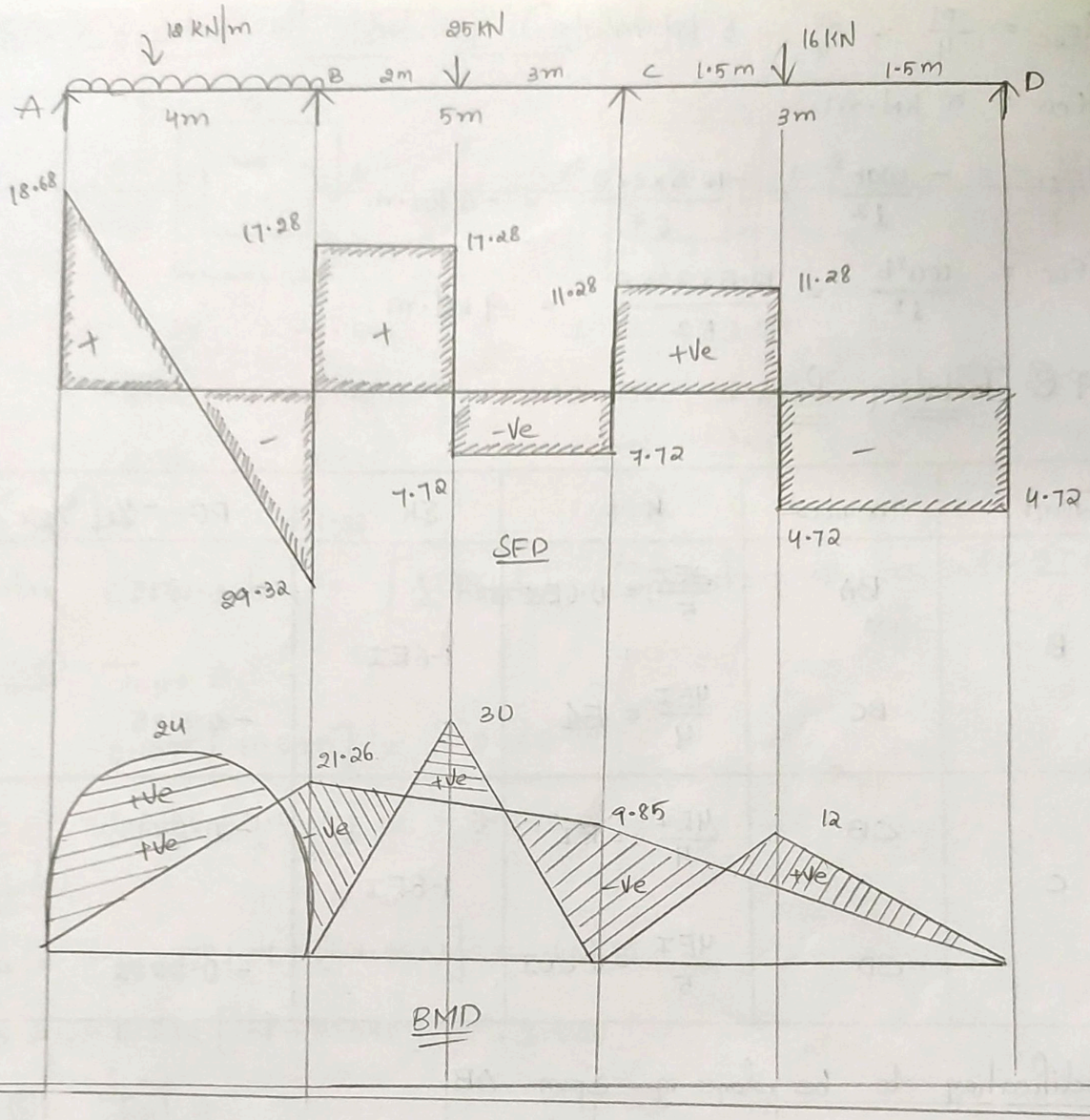
$R_{C2} \times 3 = 16 \times 1 \cdot 5 + 9 \cdot 858$

$R_{C2} = 11 \cdot 28$ kN

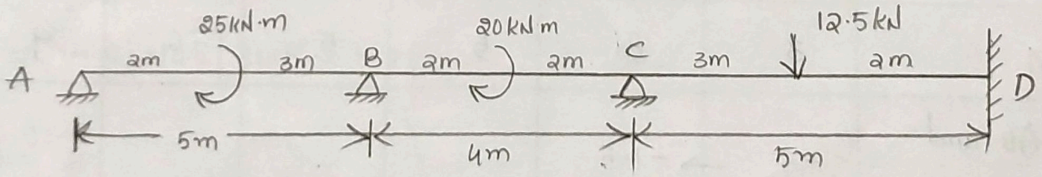
$R_D = 16 - 11 \cdot 28$

$R_D = 4 \cdot 72$ kN

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Q:- Analyse this Beam Using Kani's method:



Step ①:- Fixed End moments :

$$M_{FAB} = \frac{Mb}{l^2}(b-2a) = -\frac{25\times3}{5^2}(3-2\times2)$$

$$= 3\ kN\cdot m$$

$$M_{FBA} = \frac{Ma}{l^2}(2b-a) = \frac{25\times2}{5^2}(2\times3-2)$$

$$= 8\ kN\cdot m.$$

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$M_{FBC} = \frac{M}{4} = \frac{20}{4} = 5\ kN.m$

$M_{FCB} = 5\ kN.m$

$M_{FCD} = -\frac{wab^2}{l^2} = \frac{-12.5 \times 3 \times 2^2}{5^2} = -6\ kN.m$

$M_{FDC} = \frac{wa^2b}{l^2} = \frac{12.5 \times 3^2 \times 2}{5^2} = 9\ kN.m$

Step ② Rotation factor

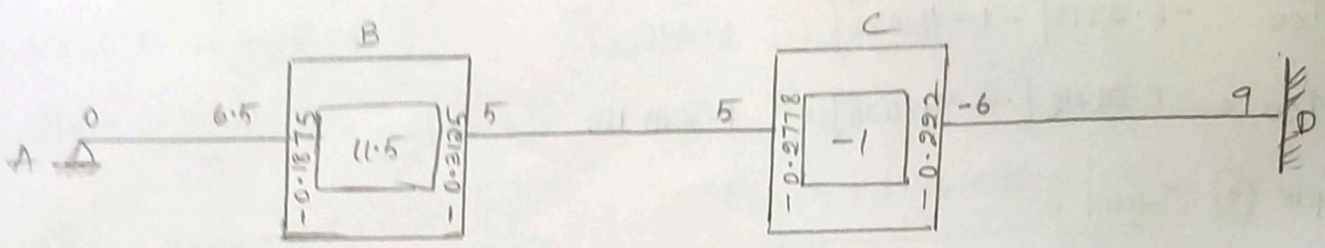
Joint	membess	k	Σk	RF $-\frac{1}{2}\left[k/\Sigma k\right]$
B	BA	$\frac{3EI}{5} = 0.6EI$	1.6EI	-0.1875
	BC	$\frac{4EI}{4} = EI$		-0.3125
C	CB	$\frac{4EI}{4} = EI$	1.8EI	-0.2778
	CD	$\frac{4EI}{5} = 0.8EI$		-0.2222

modification to be done in Span ~~AB~~

Members	~~AB~~	BA	BC	CB	CD	DC
FEM	3	8	5	5	-6	9
Release ~~AB~~ and Carry over to BA	-3	→ -1.5				
Adjusted FEM	0	6.5	5	5	-6	9

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Step ⑤:- Calculation of Rotation Contribution



-2.156	-3.593	1.275	1.020
-2.395	-3.992	1.386	1.109
-2.416	-4.026	1.396	1.116
-2.418	-4.03	1.397	1.117

Rotation Contribution = [Σ Rotation Contribution at far End + ΣFEM] × RF

Cycle ① Joint B:-

$M'_{AB} = -0.1875[11.5+0] = -2.156$

$M'_{BC} = -0.3125[11.5+0] = -3.593$

Joint C:

$M'_{CB} = -0.2778[-1+(-3.593)] = 1.275$

$M'_{CD} = -0.222[-1-3.593] = 1.020$

Cycle ② :- Joint B :-

$M'_{AB} = -0.1875[11.5+1.275] = -2.395$

$M'_{BC} = -0.3125[11.5+1.275] = -3.992$

Joint C:-

$M'_{CB} = -0.2778[-1-3.992] = 1.386$

$M'_{CD} = -0.222(-1-3.992) = 1.109$

Cycle ③ : Joint B

$M'_{AB} = -0.1875[11.5+1.386] = 2.416$

$M'_{BC} = -0.3125[11.5+1.386] = 4.026$

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Joint C:-

$M'_{CB} = -0.2778[-1-4.026] = 1.396$

$M'_{CD} = 0.222[-1-4.026] = 1.116$

Cycle (4) Joint B:

$M'_{BA} = -0.1875[11.5+1.396] = -2.418$

$M'_{BC} = -0.3125[11.5+1.396] = -4.03$

Joint C:-

$M'_{CB} = -0.2778[-1-4.03] = 1.397$

$M'_{CD} = -0.222[-1-4.03] = 1.117$

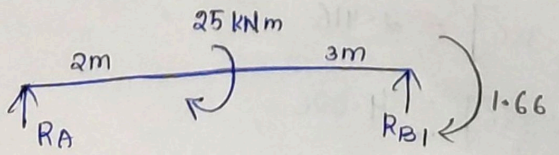
Step (4) :- Final moments

Final moments = ΣFEM's + (2 near End Contributions + far End Contributions).

Member	AB	BA	BC	CB	CD	DC
FEM	0	6.5	5	5	-6	9
2x near End Contribution	0	2x(-2.418) -4.836	2x-4.03 8.06	2x1.397 2.794	2x1.117 2.234	0
far End Contribution	0	0 1.397	1.9397 -4.03	-4.03	0	1.118
final	0	+1.66	-1.66	3.76	-3.76	10.12

Step (5) :- Shear force and BM (free Bending moments) :-

Span AB:-



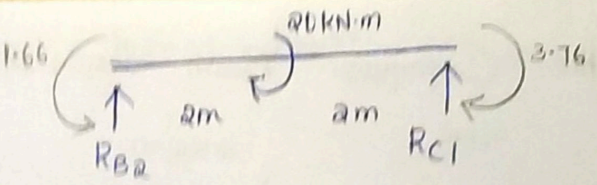
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$R_A + R_{B_1} = 0$

$R_A \times 5 + 1.66 + 25 = 0$

$R_A = \frac{-26.66}{5} = 5.33\ kN\ (\downarrow)$

$\therefore \boxed{R_{B_1} = 5.33\ kN}$

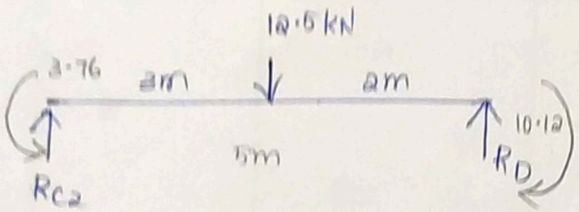


$R_{B_2} + R_{C_1} = 0$

$R_{B_2} \times 4 + 3.76 + 20 = 1.66$

$\boxed{R_{B_2} = 5.53\ kN\ \downarrow}$

$\boxed{R_{C_1} = 5.53\ kN}$



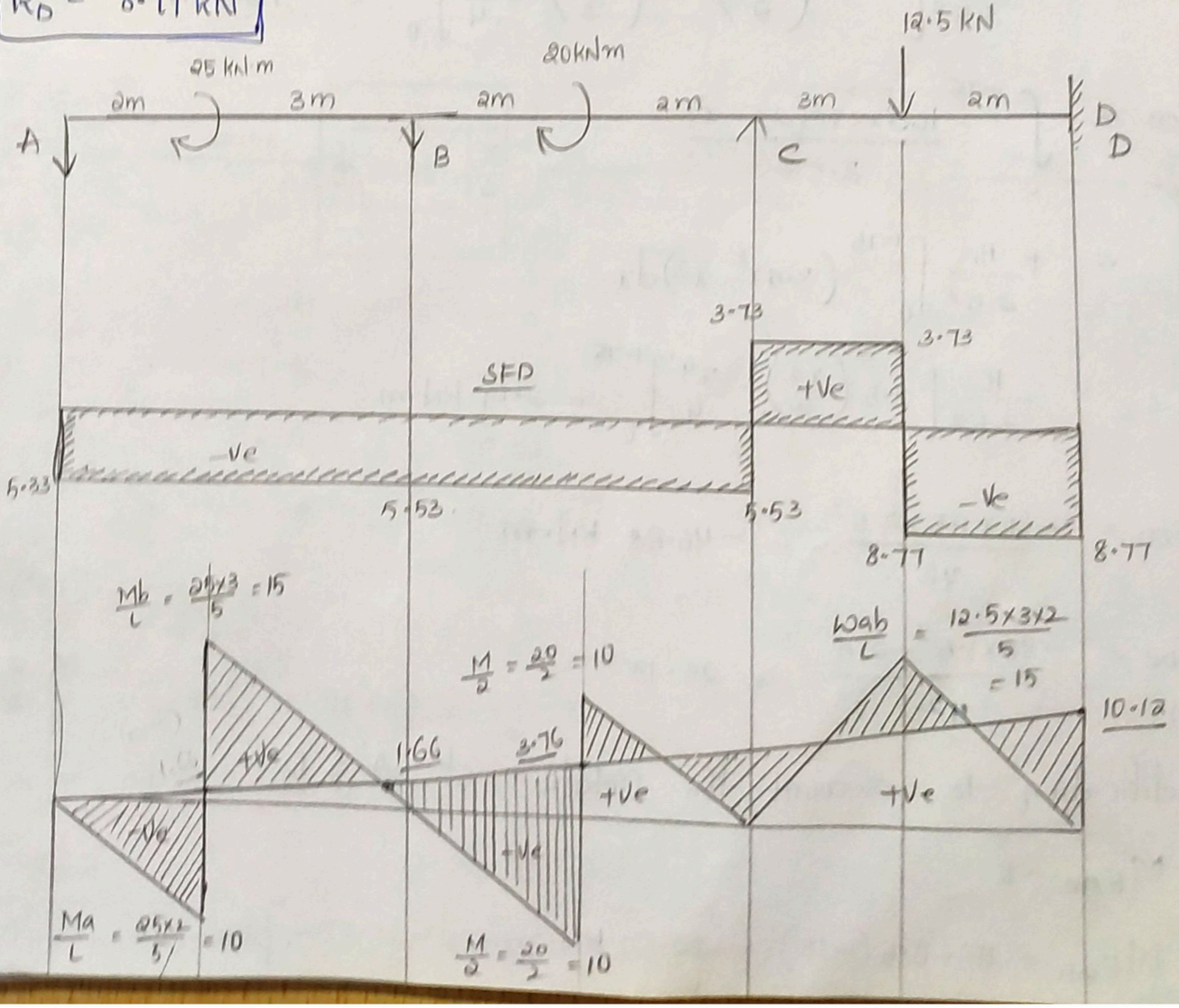
$R_{C_2} + R_D = 12.5\ kN$

$R_{C_2} \times 5 + 10.12 = 12.5 \times 2 + 3.76$

$\boxed{R_{C_2} = 3.73\ kN}$

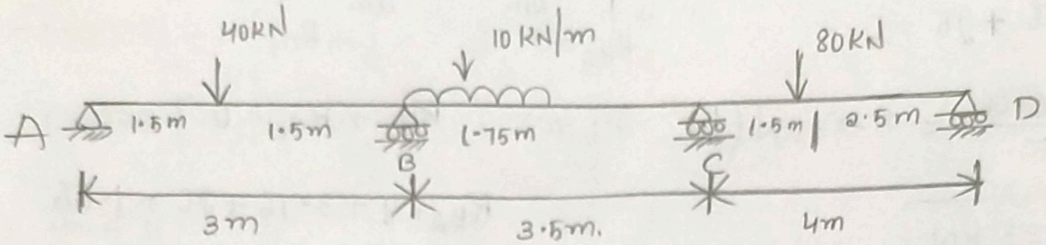
$R_D = 12.5 - 3.73$

$\boxed{R_D = 8.77\ kN}$



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Q:- Analyse the Continuous Beam shown in figure:- by Kani's method:-



Sol:- Step ①:- Fixed End moments:-

$$M_{FAB} = -\frac{40\times3}{8} = -15 \text{ KN.m}.$$

$$M_{FBA} = 15 \text{ KN-m}.$$

$$M_{FBC} = -\int_0^{1.75} \frac{10\,dx \times x(3.5-x)^2}{3.5^2}$$

$$= \frac{-10}{3.5^2}\int_0^{7.5} (3.5^2x - 7x^2 + x^3)\,dx.$$

$$= \frac{-10}{3.5^2}\left[3.5^2\left(\frac{x^2}{2}\right) - 7\left(\frac{x^3}{3}\right) + \frac{x^4}{4}\right]_0^{1.75} = -7.02 \text{ KN-m}.$$

$$M_{FCB} = \int_0^{1.75} \frac{10\,dx \times x^2(3.5-x)}{3.5^2}$$

$$= +\frac{10}{3.5^2}\int_0^{1.75} (3.5x^2 - x^3)\,dx.$$

$$= \frac{10}{3.5^2}\left[3.5\left(\frac{x^3}{3}\right) - \frac{x^4}{4}\right]_0^{1.75} = 3.19 \text{ KN.m}.$$

$$M_{FCD} = -\frac{80\times1.5\times2.5^2}{42} = -46.88 \text{ KN.m}.$$

$$M_{FDC} = -\frac{80\times1.5^2\times2.5}{42} = 28.13.$$

Modification to account for rotation of A & D

$$M_{FAB} = 0$$

$$M_{FBA} = 15 - 0.5(-15) = 22.5 \text{ KN.m}.$$

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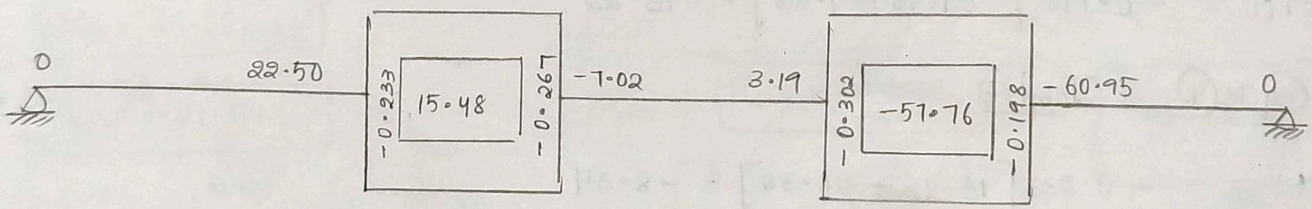
$M_{FCD} = -46.88\,(0.5 \times 28.13) = -60.95$ kN.m

$M_{FDC} = 0$

Step ②: Rotation factor:

Joint	members	K	ΣK	RF
B	BA	$\frac{3EI}{3} = EI$	2.143 EI	-0.233
	BC	$\frac{4EI}{3.5} = 1.143EI$		-0.267
C	CB	1.143 EI	1.893 EI	-0.302
	CD	$\frac{3EI}{4} = 0.75EI$		-0.198

Step ③: Calculation of rotation Contributions:



-3.61	-4.13	18.69	12.25
-7.96	-9.12	20.20	13.24
-8.31	-9.53	20.32	13.22
-8.34	-9.56	20.33	13.33

Cycle ①:- Joint B:-

$M'_{AB} = -0.233\,[15.48 + 0] = -3.61$

$M'_{BC} = -0.267\,[15.48 + 0] = -4.13.$

Joint C:

$M'_{CB} = -0.302\,[-57.76 - 4.13] = +18.69$

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$M'_{CD} = -0.198[-57.75 - 4.13] = 12.25$

Cycle ②: Joint B:-

$M'_{BA} = -0.233[15.48 + 18.69] = -7.96$

$M'_{BC} = -0.267[15.48 + 18.69] = -9.12.$

Joint C:

$M'_{CB} = -0.302[-57.76 - 9.12] = 20.20$

$M'_{CD} = -0.198[-57.76 - 9.12] = 13.24$

Cycle ③:- Joint B:

$M'_{BA} = -0.233[15.48 + 20.20] = -8.31$

$M'_{BC} = -0.267[15.48 + 20.20] = -9.53$

Joint C:

$M'_{CB} = -0.302[-57.76 - 9.53] = 20.32$

$M'_{CD} = -0.198[-57.76 - 9.53] = 13.22.$

Cycle ④: Joint B:-

$M'_{BA} = -0.233[15.48 + 20.32] = -8.34$

$M'_{BC} = -0.267[15.48 + 20.32] = -9.56.$

Joint C:-

$M'_{CB} = -0.302[-57.76 - 9.56] = 20.33$

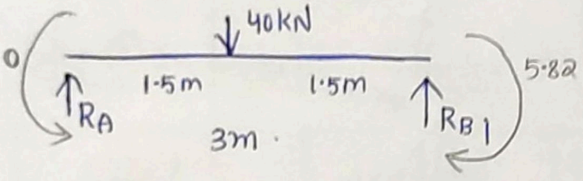
$M'_{DC} = -0.198[-57.76 - 9.56] = 13.33.$

Step ⑤: Final moment Calculations:-

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	A	B		C		D
FEM	0	22.5	-7.02	3.19	-60.95	0
2x near End Contributions	2x0 = 0	2x-8.34 = -16.68	2(-9.56) -19.12	2(20.33) 40.66	2(13.33) = 26.66	2x0 0
Far End Contribution	0	0	20.33	-9.56	0	0
Final moments	0	5.82	-5.82	34.29	-34.29	0

Step ⑤: Shear force and Bree Bending moments.

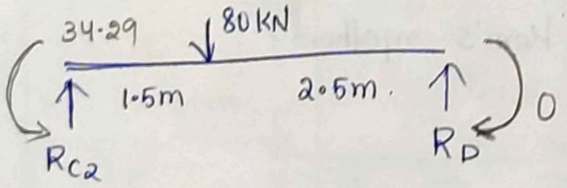


$R_A + R_{B_1} = 40$

$R_A \times 3 + 5.82 = 40 \times 1.5 + 0$

$R_A = 18.06$ kN

$R_{B_1} = +21.94$ kN

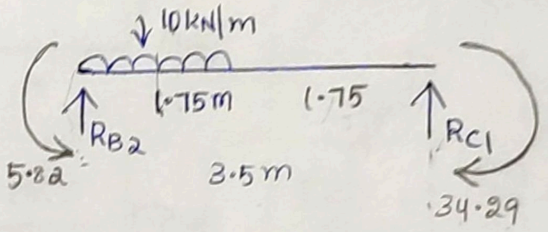


$R_{C2} + R_D = 80$ kN

$R_{C2} \times 4 = 80 \times 2.5 + 34.29$

$R_{C2} = 58.57$ kN

$R_D = 21.42$ kN



$R_{B_1} + R_{B_2} = 10 \times 1.75$

$R_{B_2} \times 3.5 + 34.29 = 5.82 + \left(10 \times 1.75 \times \frac{1.75}{2}\right)$ +1.75

$R_{B_2} = -3.75$ kN +4.990

$R_{B_2} = 4.990$ kN

$R_{C_1} = +3.75 - 17.5$ 4.990

$R_{C_1} = 21.25$ kN

$R_{C_1} = 12.51$ kN

Free Bending moments

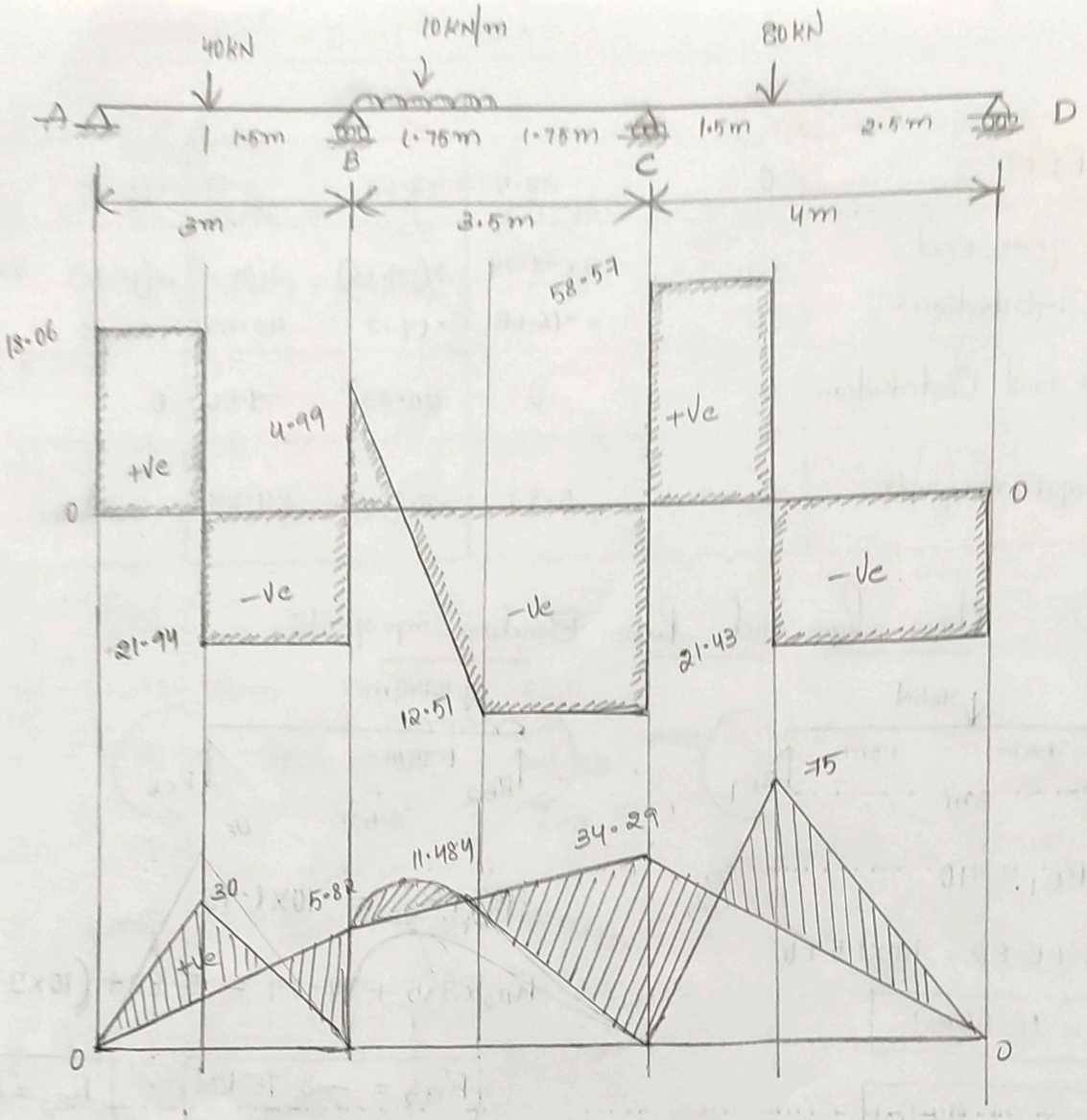
Span AB $= \frac{wl}{4} = \frac{40 \times 3}{4} = 30$ kN·m

Span BC $= \frac{wab}{l} = \frac{(10 \times 1.75)(0.875)(2.625)}{3.5}$ 11

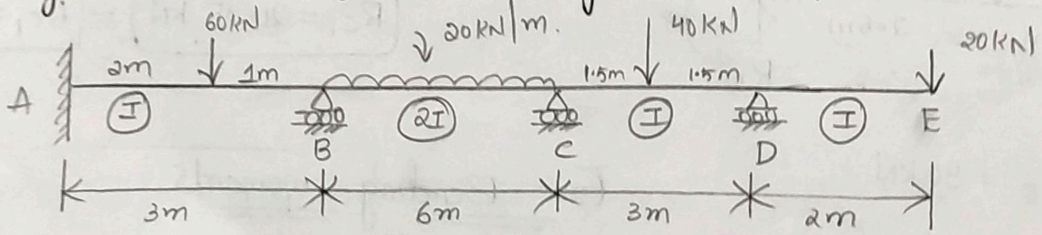
Span CD $= \frac{wab}{l} = \frac{80 \times 1.5 \times 2.5}{4}$ 11.484 kN·m

$= 75$ kN·m

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Q:- Analyse the Continuous beam by Kani's method.



Sol: Step ① :- Fixed End moments :-

$$M_{FAB} = \frac{-wab^2}{l^2} = \frac{-60\times2\times1^2}{3^2} = -13.33 \text{ KN-m}.$$

$$M_{FBA} = \frac{wa^2b}{l^2} = \frac{60\times2^2\times1}{3^2} = 26.67 \text{ KN-m}.$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-20\times6^2}{12} = -60 \text{ KN-m}$$

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$M_{FCB} = 60$ kN·m

$M_{FCD} = -\frac{40 \times 3.0}{8} = -15$ kN·m

$M_{FDC} = 15$ kN·m

$M_{FDE} = -2 \times 20 = -40$ kN·m

⇒ Modification in FEM to account for rotation of D

$M_{FDC} = -40$ kN·m (to balance joint C)

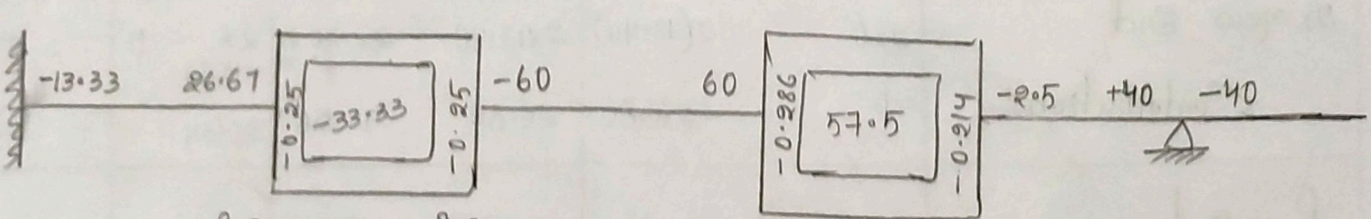
$M_{FCD} = -15 - 0.5(15 - 40) = -2.5$ kN·m

Rotation factor $= -\frac{1}{2}\left[\frac{K}{\Sigma K}\right]$

Step ②:- Rotation factor :-

Joint	members	K	ΣK	RF $-\frac{1}{2}\left[\frac{K}{\Sigma K}\right]$
B	BA	$\frac{4EI}{3}$	$8/3\,EI$	-0.25
	BC	$\frac{4E(2I)}{6} = \frac{4}{3}EI$		-0.25
C	CB	$\frac{4E(2I)}{6} = \frac{4}{3}EI$	$2.33\,EI$	-0.286
	CD	$\frac{3EI}{3} = EI$		-0.214

Step ③: Calculation of rotation Contribution :-



8.33	8.33	−18.83	−14.09	
13.04	13.04	−20.17	−15.10	
13.38	13.38	−20.17	−15.17	
13.40	13.40	−20.28	−15.17	

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Cycle ① Joint B

$M'_{AB} = -0.25[-33.33+0] = 8.33$

$M'_{BC} = -0.25[-33.33+0] = 8.33$

Joint C :-

$M'_{CB} = -0.286[57.5+8.33] = -18.83$

$M'_{CD} = -0.214[57.5+8.33] = -14.09$

Cycle ②: Joint B :-

$M'_{BA} = -0.25[-33.33-18.83] = 13.04$

$M'_{BC} = -0.25[-33.33-18.83] = 13.04$

Joint C :-

$M'_{CB} = -0.286[57.5+13.04] = -20.17$

$M'_{CD} = -0.214(57.5+13.04) = -15.10$

Cycle ③ Joint B

$M'_{BA} = -0.25[-33.33-20.17] = 13.38$

$M'_{BC} = -0.25[-33.33-20.17] = 13.38$

Joint C

$M'_{CB} = -0.286(57.5+13.38) = -20.27$

$M'_{CD} = -0.214(57.5+13.38) = -15.17$

Cycle ④ Joint B

$M'_{BA} = -0.25(-33.33-20.27) = 13.40$

$M'_{BC} = -0.25(-33.33-20.27) = 13.40$

Joint C :-

$M'_{CB} = -0.286(57.5+13.40) = -20.28$

$M'_{CD} = -0.214(57.5+13.40) = -15.17$

Step ④: Final moment Calculations

Members	A	B		C		D	E
FEM	-13.33	26.76	-60	60	-2.5	40	-40	0
2x near End Contributions	2x0 =0	2(13.40) = 26.80	2x13.40 26.80	2x-20.28 40.56	2x -15.17 30.34	0 –	– –	– –
far End Contributions	13.40	0	-20.28	13.40	0	–	–	–
Final moments.	0.07	53.47	-53.47	32.84	-32.84	40	-40	0

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